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Implementation of Machine Learning in Structural Reliability Analysis

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ABSTRACT

Reliability is a probabilistic measure of structural safety. In Structural Reliability Analysis (SRA), both loads and resistances are modelled as probabilistic variables, and the failure of structure occurs when the total applied load is larger than the total resistance of the structure. The probability distribution of the loads as well as the resistance can depend upon multiple variables. Considering all these factors, the probability of failure of a structure is calculated. SRA can be used for systematic adjustment of structural safety factors, and for the probabilistic design and operation of structures. For example, SRA can be used to design a structure to operate during the desired lifetime safely, or it can be used for maintenance scheduling of structural systems to prevent potential failures.

Machine learning (ML) is the study of computer algorithms that can improve automatically through experience and by the use of data. Machine learning and statistics are closely related fields in terms of methods, but distinct in their principal goal: statistics draws inferences from a sample, while machine learning finds generalizable predictive patterns. ML methods can be applied to analytical and numerical SRA methods, such as First/Second-Order Reliability Methods (FORM/SORM) and First Order Second Moment (FOSM)

Keywords: probabilistic models, Structural Reliability Analysis, Machine learning

1. STRUCTURAL RELIABILITY ANALYSIS

Structural Reliability Analysis is typically performed by calculation or prediction of the probability of violating a performance function of a system during its life. This probabilistic approach yields a generalized reliability formulation for many SRA problems, as presented in :

$$P_f(t) = P\big[G\big(x(t)\big) \leq 0\big] = \int_{G(x(t))\leq 0} f_{x(t)}\big(x(t)\big)dx(t)$$

where,

$P_f(t)$: The probability of failure

$f_{x(t)}(x(t))$: The joint Probability Density Function (PDF) of vector of random variables $x(t)$

$G(x(t))$: the defined performance measure known as Limit State Function (LSF)

This is a generalized equation and different SRA approaches can be derived from it. For example, limiting t to a specific instant or assuming the system to be time-invariant, will result in a time-independent reliability equation, as follows:

$$P_f = P[G(x) \leq 0] = \int_{G(x)\leq 0} f_x(x)dx$$

1.1. Structural Reliability Analysis Methods

SRA methods based on Taylor series expansion like first and second order reliability methods (FORM/SORM) are used to provide an analytical solution for the reliability equation. However, they require information about the LSF and its derivatives, which may require significant computational power. Another tool for structural reliability evaluation is the Monte Carlo Simulation, which is known as an approach with a large computational cost.

1.1.1. First/Second-Order Reliability Methods

The computation of P_f by Eq. is called the full distributional approach and can be considered to be the fundamental equation of structural reliability analysis. In general, the joint probability density function of random variables is practically impossible to obtain. Even if this function is available, the evaluation of the integral is extremely complicated. Therefore, one possible approach is to use analytical approximations of this integral that are simpler to compute. For clarity of presentation, all these methods can be grouped into two types, namely, first- and second-order reliability methods (FORM and SORM).

The limit state functions of interest can be linear or nonlinear functions of the basic variables. FORM can be used to evaluate the equation when the limit state function is a linear function of uncorrelated normal variables or when the nonlinear limit state function is represented by the first-order (linear) approximation. The SORM estimates the probability of failure by approximating the nonlinear limit state function, including a linear limit state function with correlated nonnormal variables, by a second-order representation.

1.1.2. First Order Second Moment Method

The First Order Second Moment method derives its name from the fact that it is based on a first-order Taylor series approximation of the performance function or limit state function and uses only second moment statistics (means and covariances) of the random variables.

Consider the limit state function,

$$G(x_1, x_2, \dots, x_n) = a_0 + a_1x_1 + a_2x_2 + \cdots + a_nx_n$$

Reliability index can be obtained by the following equation:

$$\beta = a_0 + \frac{\sum_{i=1}^{n} a_i\mu_{x_i}}{\sqrt{\sum_{i=1}^{n} a_i^2\sigma_{x_i}^2}}$$

The above reliability index definition depends only on the mean and standard deviation of the random variables.

2. MACHINE LEARNING

Machine Learning is a subset of artificial intelligence (AI) and uses computer algorithms to perform a specific task by the use of data without giving explicit instructions. Based on the availability of the sample data, ML methods can be categorized into supervised, unsupervised, semi-supervised, active, and reinforcement learning algorithms. In simple words, supervised learning algorithms are used when the sample data contain input-output pairs, called labelled data, obtained from the desired function or distribution. When the available data only consists of input values, unsupervised learning algorithms are mostly applied, and in cases in which some of the sample data lack the output value, semi-supervised algorithms are mostly utilized. Active Learning (AL) methods can be considered as a special case of semi-supervised learning, where some of the data are actively chosen to obtain the output value. For reinforcement learning, the output is given as reward feedback to the actions taken based on the selected inputs. It is important to select a proper learning approach based on the available data. As mentioned, ML methods are being more commonly used as surrogate models of the structural response under deterministic or stochastic loading conditions. In other words, based on the available data, they can estimate the performance function (i.e., the structural response) or approximate the LSF. In the first case, the gathered data contain the variables and the corresponding values of the performance function, regardless of the failure criteria. After training the ML-based model with the available data, the model will be employed to estimate the performance function at the desired points. The result is then used to calculate the LSF, and finally, the probability of failure. In the second case, having the LSF values for each set of variable data, the ML-based model will estimate the LSF at the desired points directly. Then, the failure probability is obtained using one of the calculation methods, such as FORM, SORM, FOSM, and MCS. In addition to PF and LSF, reliability index and failure probability have been estimated using ML methods. Nevertheless, generic ML methods need specific considerations to be used in SRA applications. The essential elements for an ML algorithm can be classified as (I) training and testing data, (II) an objective function, (III) an optimization algorithm, and (IV) a model to be selected for the data. In many ML applications, we usually have a proper amount of training data to be used for making the ML model. However, in SRA problems, we typically deal with the probability of rare events, which leads to a marginal amount of training data. Moreover, the model to be nominated for the data (such as linear, nonlinear, nonparametric, etc.) needs to be accurately selected. The latter facts make the use of the ML-based method challenging in some respects. The process and special considerations for each ML-based model and reliability calculation are further elaborated in the respective sections of this paper.

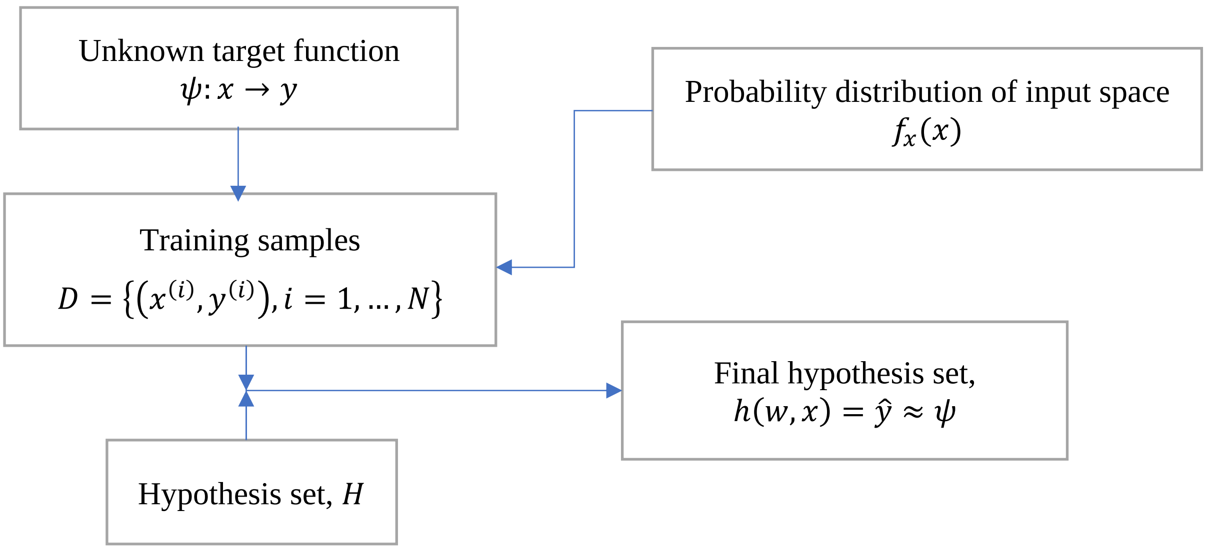

Figure 1. Schematic diagram of supervised learning

3. MACHINE LEARNING IN STRUCTURAL RELIABILITY ANALYSIS

Structural reliability analysis methods based on some of the most common machine learning algorithms are explained below.

3.1. Artificial Neural Network based SRA

Artificial Neural Networks (ANN) are amongst the most popular machine learning methods. ANNs are capable of adapting to parallel distributed processing and being applied in large-scale multi-dimensional problems. A well-trained ANN, either with the classification or regression approach, can model highly nonlinear problems accurately and efficiently. With the classification approach to the SRA problems, the ANN is used to identify whether the structure operates within the safe domain or not. On the other hand, the regression approach is used to estimate the value of an implicit function. To this extent, the most common application of ANNs with the regression approach in the SRA studies is modelling the LSF or the Performance Function (PF) to be combined with reliability calculation models such as MCS, FORM, SORM, and FOSM. For this purpose, the input variables are usually the structure properties, design variables, and operational conditions such as material properties, load fluctuations, and geometric properties. The output can be the value of LSF, the performance function, or the reliability index. Also, in problems with more than one LSF and multiple failure modes, ANNs with multiple outputs are used. However, it is worthwhile to consider the required training and analysis time when using ANN to approximate the LSF in an SRA problem. A helpful criterion for processing time is given in Eq.

$$nT^{E} \gg (NT^{G}) + (NT^{E}) + T^{T} + (nT^{NA})$$

where n is the total number of required exact reanalysis and T^{E} is the time for each analysis. Then, given a network architecture, N is the total sample size, T^{G} is the time to generate each sample, T^{T} is the network training time, and T^{NA} is the required reanalysis time by the trained network.

The most common ANNs in the SRA applications are the feedforward Multi-Layer Perceptron (MLP) and Radial Basis Function (RBF) networks. The primary processing unit of an ANN is a single neuron that receives inputs through its axon connections. The input data are then processed to beget the desired output. This process in the aforementioned ANNs is carried out in two steps, as shown in Fig. 2. In MLP (Fig. 2), in the first step, a weighted sum of the inputs is calculated. Formally, a constant "threshold" value, $x_0 = 1$, is added as an additional input. The scalar weighted sum is written as $\boldsymbol{w}^{T}\boldsymbol{x} + w_0$, where $\boldsymbol{w}$ is the weight vector and $\boldsymbol{x}$ is the input vector. In the second step, the resulting sum is fed into an "activation "or "transfer" function to calculate the final output. Several commonly used activation functions are: linear function ($f(z) = z$), signal or sign function ($f(z) = SIGN(z)$), logistic sigmoid ($f(z) = \frac{1}{1+e^{-\gamma z}}$), unit step ($f(z) = H(z - z_0)$), hyperbolic tangent ($f(z) = \tanh(z)$), orthogonal polynomials, etc.

In RBF networks (Fig. 2), in the first step, the Euclidean Norm of the input vector and a central vector $\boldsymbol{t}$ is calculated and multiplied by a threshold, b, as $\|x - t\|.b$. In the second step, a radial basis function, such as the Gaussian function, acts on the calculated value to obtain the neuron output s.

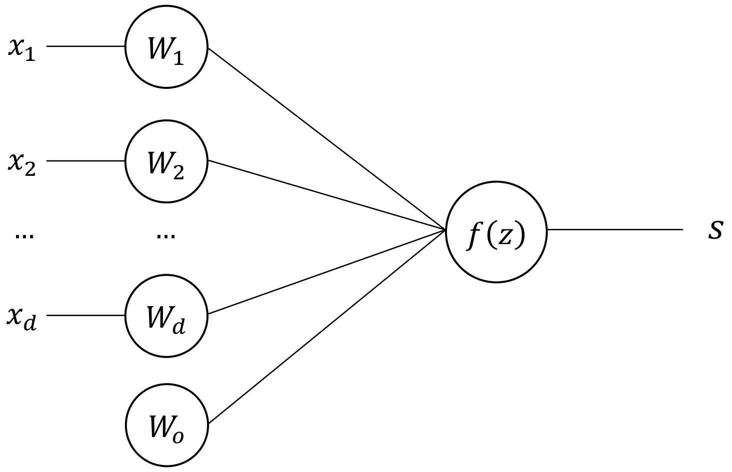

Figure 2. Single neuron unit in an MLP structure

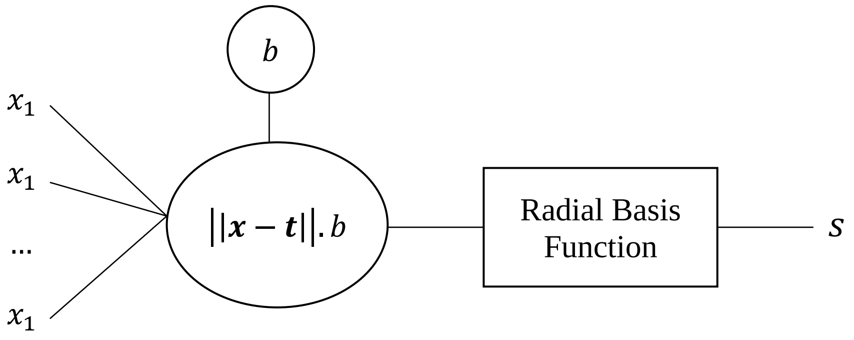

Figure 3. Single neuron unit in an RBF structure

In the common RBF and MLP structures, neurons form fully interconnected layers, as shown in Fig. 3 In other words, all neurons in each layer are connected to all neurons in the previous and next layers. There are no connections within each layer in these structures. In general, MLP achieves a better global approximation of the designated output, while RBF networks have better local approximations as compared to MLP. Moreover, RBF networks require less learning.

3.2. Support Vector Machine based SRA

SVMs are widely used to perform classification and regression tasks in the SRA. Some reviews of the SVM-based SRA methods have been presented in [1-6]. They are particularly suitable for high-dimensional problems. Their concept is defined based on structural risk minimization, in contrast with ERM. Hence, they result in good generalization [3]. In the SRA, SVMs with different modifications have been generally employed as surrogate models of the LSF or the PF. The model is then used in combination with an SRA method, such as FORM, SORM, FOSM, or MCS. Here, a brief general description of SVMs is given in section 2.1. Section 2.2 summarizes the different approaches to SVM-based SRA.

The main idea behind an SVM is to find the hypothesis with the maximum distance (margin) from the closest data. For a linear SVM classifier (SVC), the hypothesis is in the form of a hyperplane in the feature space, defined by $\boldsymbol{w}^T\boldsymbol{x}+w_0=0$, where $\boldsymbol{w}$ is the weight vector of the hyperplane. In SRA, this hyperplane corresponds to the LSF, separating the failure and safe operation domains. Given the training set $D=\{(\boldsymbol{x}^{(n)},\boldsymbol{y}^{(n)}),n=1,\dots,N\}$, the objective optimization problem is in the form of $J(\boldsymbol{w})=\min\frac{1}{2}||\boldsymbol{w}||^2$, subject to $|\boldsymbol{w}^T\boldsymbol{x}^{(n)}+w_0-y^{(n)}|\leq\varepsilon$. This is rewritten as Eq. (11) in the form of the polynomial Lagrange duality problem.

$$\max_{\alpha\geq 0}\left(\sum_{n=1}^{N}\alpha_n-\frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}\alpha_n\alpha_m y^{(n)}y^{(m)}x^{(n)^T}x^{(m)}\right)$$

where $\alpha_n, n=1,\dots,N$ are the Lagrange coefficients of the cost function. The vector $\boldsymbol{\alpha}$ will be obtained with some zero value elements via solving the above optimization problem. The points $\boldsymbol{x}^{(n)}$ with corresponding $\alpha_n > 0$ are called the support vectors. These points are located on the marginal hyperplane and are used to calculate the weight vector as $\boldsymbol{w} = \sum_{\alpha_n>0} \alpha_n\, y^{(n)}x^{(n)}$. It is shown that the number of support vectors in SRA is generally small. In other words, the number of required samples for training is small. Also, the number of learning parameters in this formulation is related to the number of samples instead of the number of features. Hence, for problems with high dimensions, this method is computationally less expensive.

3.3. Bayes' Theorem based SRA

The Bayes' theorem is considered as a probability or statistics theory that describes the probability of an event using conditional probabilities and prior records associated with an event such as failure. Generally, for any probability distribution over the two events, A and B, the Bayes' theorem is stated as:

$$P(A|B)=\frac{P(B|A)}{P(B)}P(A)$$

Based on Bayes' rule, different concepts have been developed, and it is important to know their differences before having a survey on the Bayesian learning-based SRA methods. The most common forms of using Bayesian-based methods in SRA problems can be categorized as follows:

Bayesian Inference: the term "Bayesian Inference" (BI) is a general term that is usually used when the Bayes' rule is applied to update the probability of some parameters using given data, for example, $P(A)$, in Eq. can represent the observed data. Bayesian Regression: Bayesian Regression (BR) generally can be described as the utilization of BIs on regression models. In other words, the BR is a way of finding the posterior probability of an event based on several weights as in Eq.

Naïve Bayes: Finding $P(A|B)$ while assuming an independency between the measurement features is called 'naïve' Bayesian assumption. The naïve Bayesian is commonly used for classification purposes; the well-known term, naïve Bayesian classifier (NBC), is used in such cases. Bayesian Network: A Bayesian Network (BN) is a graphical description of conditional probabilities. Most of the Bayesian inferences such as BR, naïve Bayesian, etc., can be written in the form of a BN.

3.4. Kriging Estimation for SRA

This section reviews the application of Kriging estimation with an active learning perspective to be used in the SRA problems. The active learning methods presented in this section have been widely used in Kriging-based methods. However, they can be used with other SRA methods, and their application is not only limited to Kriging-based methods. Kriging or Gaussian process modelling is a common meta-modelling method that is widely used in SRA literature.

Eq. below represents a model which is proposed by Sacks et al. [2], and it is considered as a common Kriging model for estimating the response $\mathbf{Y}(\mathbf{x})$

$$\boldsymbol{Y}(\boldsymbol{x})=\boldsymbol{f}^{\boldsymbol{T}}(x)\boldsymbol{\beta}+\boldsymbol{Z}(\boldsymbol{x})$$

where $\boldsymbol{f}(\boldsymbol{x})$ is a known function of $\boldsymbol{x}$, and $\boldsymbol{\beta}$ is a vector of regression coefficients. In addition, $\boldsymbol{Z}(\boldsymbol{x})$ is assumed as a Gaussian stationary process to compensate for the deviation from $\boldsymbol{f}^{\boldsymbol{T}}(\boldsymbol{x})\boldsymbol{\beta}$. To construct an accurate Kriging model, the functional basis of the Kriging model, $\boldsymbol{f}(\boldsymbol{x})$, needs to be nominated properly.

4. CONCLUSION

Structural Reliability analysis is one of the prominent fields in civil engineering. However, an accurate SRA in most cases deals with complex and costly numerical problems. The need for cost-effective structures has encouraged researchers to look for a solution to increase the accuracy of SRA methods. Machine learning-based techniques have been introduced to the SRA problems to deal with this huge computational cost and increase accuracy. ANNs and SVMs are two popularly used tools in the ML-based SRA literature. They have been widely used for the SRA because of their adaptability to different well-known reliability calculation methods such as FORM, SORM, and MCS. Moreover, Bayes' theorem as a probability theory has also been utilized for some important SRA applications, such as updating the SRA model, constructing regression models for finding the posterior probability of events, and Bayesian networks and naïve Bayes classifiers for finding the probability of failure. Kriging Estimation is another technique that has a prominent potential in the calculation of limit state functions for further SRA. As the demand for precise and efficient SRA methods increases, the need for different artificial intelligence-based techniques like machine learning is on the rise.

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