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Effect of soil behavior model on drilling response of anchor-reinforced excavation

Mehran Javanmard ¹, Mehdi Noruzi ^{2*}

¹ Department of Civil Engineering, Zanjan University, Zanjan, Iran.

² Department of Civil Engineering, Islamic Azad University, Zanjan Branch, Zanjan, Iran.

*Correspondence should be addressed to Mehdi Noruzi, Department of Civil Engineering, Islamic Azad University, Zanjan Branch, Zanjan, Iran. Tel: +989121417531; Fax: +982433448007; Email: nmhd097@gmail.com

ABSTRACT

Although The reinforced elements such as nailing and anchor have been widely used for the stability of excavation and trench because of not taking up a large space, improved soil properties by injection, greater safety, and the possibility of being used as a permanent retaining structure. Due to the complex behavior of reinforced excavation, the stability analysis of reinforced excavation is performed by the finite element method. Some factors such as boundary interval, dimensions and type of elements, and type of behavior model of materials affect the numerical results. Due to the complex behavior of the soil stress-strain, influence from stress path and loading history, and the existence of groundwater, different behavior models have been proposed to simulate the materials. In this study, the effect of the soil behavior model on the response of anchored excavation was investigated. For this purpose, using the finite element method in the plane strain conditions, the excavation reinforced with the anchorage system was simulated for different geometrical conditions, and the results of the excavation response were compared for the Mohr-Coulomb, Drucker-Prager, and modified Cam-Clay behavior models. In the shallow excavation, it was found that the Mohr-Coulomb behavior model has the least displacement, and the Drucker-Prager behavior model has the largest lateral displacement. The Drucker-Prager behavior model should be considered as a reliable criterion for the design and control of the excavation because of the greater results regarding the lateral displacement of excavation and generally, excavation deformation.

Keywords: reinforced excavation, lateral displacement, Mohr-Coulomb, Drucker-Prager, Cam-Clay

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1. INTRODUCTION

The excavation, design, and construction of retaining structures have been widely discussed in civil engineering and need to be explored and studied in terms of geotechnics, structure, materials, technology, implementation, and economic and social considerations. As a result, it can be stated that the selection of the appropriate method for solving the problems caused by the excavation depends on all the effective conditions and can be adopted in different ways under different conditions [1].

Anchorage is one of the important methods for the construction of retaining structures. This method is very similar to the nailing method, but it uses the strands, pre-tensioned tendons, and pre-tensioned cables instead of the reinforcement [2]. Sometimes, the pre-tensioned reinforcement can be used instead of strands. Unlike the nailing method in which the nails are not subjected to any force during the operation, the strands are subjected to tension in the anchorage method. The major advantage of

this method over the nailing method is that the soil deformation is much lower in this method, and thus, it is generally more appropriate to use this method in the excavations adjacent to the building [3]. Anchorage is used in various construction operations such as mountain cutting during the road construction, widening of the roads under the bridge piers, repair and reconstruction of old retaining systems, and temporary or permanent drilling in the urban area. Thus, on the one hand, due to the special importance of protecting the excavation, the safe and sustainable design of this method is essential, and, on the other hand, given the growing application of this method and its high operational volume, achieving the optimal and safe design of this system is of particular importance [4-5]. Due to the complex behavior of the anchor-reinforced excavation, the analysis of these structures is performed using the robust finite element method. Various factors such as boundary interval, meshing, type of element used, and behavior model of materials are effective in improving the accuracy of the numerical modeling. The soil is modeled with the simplifying assumptions due to the complex stress-strain behavior, heterogeneity and, anisotropy, influence from stress path and loading history, etc. [6]. Szavits-Nossan and Sokolić et al. (2009) compared the simulated results using Plaxis 2D software with the recorded field reports to evaluate the impact of soil behavior model type on the response of the excavation stabilized with the anchorage system. Based on the results, the use of behavior model calibration and sensitivity analysis of retaining structure simulation is effective and necessary for the improvement of numerical results, and it is recommended to conduct more extensive studies [7]. Han and Elliott et al. (2011) investigated two modeling methods of the anchorage system with the results of the instrumentation for the anchor-reinforced excavation and rock bolt systems. They evaluated both elastoplastic modeling and shear strength reduction methods. Based on the results of the bending moment distribution and lateral earth pressure, the elastoplastic method is not capable of providing a good prediction of the values [8]. Rashidi and Torabipour (2018) modeled the response results of the excavation reinforced with nailing and the pile-anchor stabilized system and the anchorage system using the numerical modeling and compared the results. In this research, to obtain the optimal numerical model, the boundary interval analysis,

dimensional mesh analysis, and element type analysis were performed. Based on the results, according to the FHWA recommendation for the minimum safety factor, the use of anchorage system resulted in less displacement than the nailing system for both types of soil. The minimum settlements behind the excavation wall were reported for the pile-anchor system that also improved the wall displacement and base rotation [9]. Nakai & Okuda (2014) evaluated the performance response of anchor-reinforced excavation using the numerical model in the plane strain environment with the experimental results. In this study, they used Plaxis software and the subloading t_{ij} model for the stepwise simulation. This behavior model considers the effects of principal stresses on the soil deformation and strength, effect of stress path on the plastic flow, and the effect of confining stress and the soil swelling and compaction. Based on the results of the physical and numerical model, the described simulation method with high accuracy can be used for modeling [10]. So far, the researchers have conducted various studies on the impact of the behavior model on the results of geotechnical problems. Han (2008) determined the required parameters of the developed Drucker-Prager behavior model for the two loading conditions of strain control parallel and perpendicular to the sample axis using the recursive method with the simulation of the triaxial test. The Drucker-Prager yield criterion is defined as the second invariant of the deviator stress tensor (J_2) and the first invariant of the stress (J_1). In this study, the parameters of this behavior model were determined for different samples under different stress paths [11]. Hofstetter et al. evaluated the sensitivity of the tunneling numerical results in two-dimensional space of plane strain to the type of selected behavior model of soil. In this study, they used five elastic, Mohr-Coulomb, Drucker-Prager, and Cam-Clay models to introduce soil stress-strain behavior. According to the results, the numerical analysis based on simple assumptions on the soil stress-strain behavior cannot simulate the soil response in tunneling operation with good accuracy, and the sensitivity analysis of soil behavior model in tunneling is of particular importance. In this model, the use of the developed Drucker-Prager behavior model (cap behavior model) provides more accurate results than the Drucker-Prager and Cam-Clay behavior models [12].

2. RESEARCH METHODOLOGY

In this study, the evaluation of the effect of soil behavior model on the drilling response of the excavation stabilized by the anchorage system was investigated and evaluated based on the software and finite element numerical methods and the ABAQUS finite element software. The behavior models used in this study are Mohr-Coulomb, Drucker-Prager, and Cam-Clay [13-15]. The Mohr-Coulomb

behavior model is a well-known model in which five parameters are considered, including two parameters for soil elasticity, namely modulus of elasticity and Poisson's ratio, and three parameters for soil plasticity, namely cohesion, internal friction angle, and dilation angle. This model is the most well-known and most common and somehow, the simplest soil behavior model, which is

suitable for initial estimation and preliminary analysis of all problems [16, 17]. The Drucker-Prager model in ABAQUS software consists of three types of linear, exponential, and hyperbolic models. In this study, the linear model is considered because of its relationship with the parameters of the Mohr-Coulomb behavior model [17-19]. The Cam-Clay soil model assumes that the voids between the solid particles are only filled with water. Significant irreversible (plastic) volume changes occur when the soil is loaded due to the water leaving the pores. The realistic prediction of the deformations is crucial for many geotechnical engineering problems. The modified Cam-Clay model formulation is based on the plasticity theory, which makes it possible to realistically predict the volume changes due to the different types of loading [17-20]. To investigating the effects of depth on the results, the excavations of 3, 8, and 14m depth were used representing the shallow, medium, and deep excavations, respectively. In these cases, the vertical

distance of the nails was assumed to be 1 m, and the inclination angle was 15°. In the other part of the study, the inclination angles of the anchor installation were assumed to be 0, 10, and 15°, and its effect on the results was investigated. Also, the lengths equal to 0.5, 0.7, and 1 times the excavation height were considered to investigate the effect of length on the results, and the parametric studies on different anchor lengths were considered. The soil density was assumed to be 17 kN/m³. The friction angle and cohesion of the soil were also assumed to be 26° and 35 kN/m², respectively. The ordinary concrete with usual specifications was used for the grouting. The usual type of steel was also considered. For different soil behavior models, the Mohr-Coulomb model was converted to other behavior models using the existing relations, usual soil properties, and the equivalent parameters were obtained. Some key parameters and the specifications needed to analyze the samples are shown in Table 1.

Table 1. Specifications of parameters and used materials

Parameter Name	Value	Unit or relative description of relevant quantity
Depth of excavation	3, 8 and 14	m
Anchor inclination angle	0, 10 and 15	degree
Length of anchor	0.5 and 0.7 and 1	equal to excavation height
Soil density	17	kN/m ³
Internal friction angle (Mohr-Coulomb model)	26	degree
Soil cohesion	35	kN/m ²
Poisson's ratio	0.35	-
Internal friction angle (Drucker-Prager model)	45	degree
Flow stress ratio for Drucker-Prager model	0.788	-
Dilation angle (Drucker-Prager model)	0.1	degree
Lambda (bulk modulus in Cam-Clay behavior model)	0.174	-
M (stress ratio in Cam-Clay behavior model)	0.94	-
Anchor density	7850	kg/m ³
Grout density	2400	kg/m ³

To verify the modeling process and the software results, the modeling performed in this study was compared by the

model of Rashidi et al. [21]. Figure 1 and Table 1 illustrate the parameters used in the model of Rashidi et al.

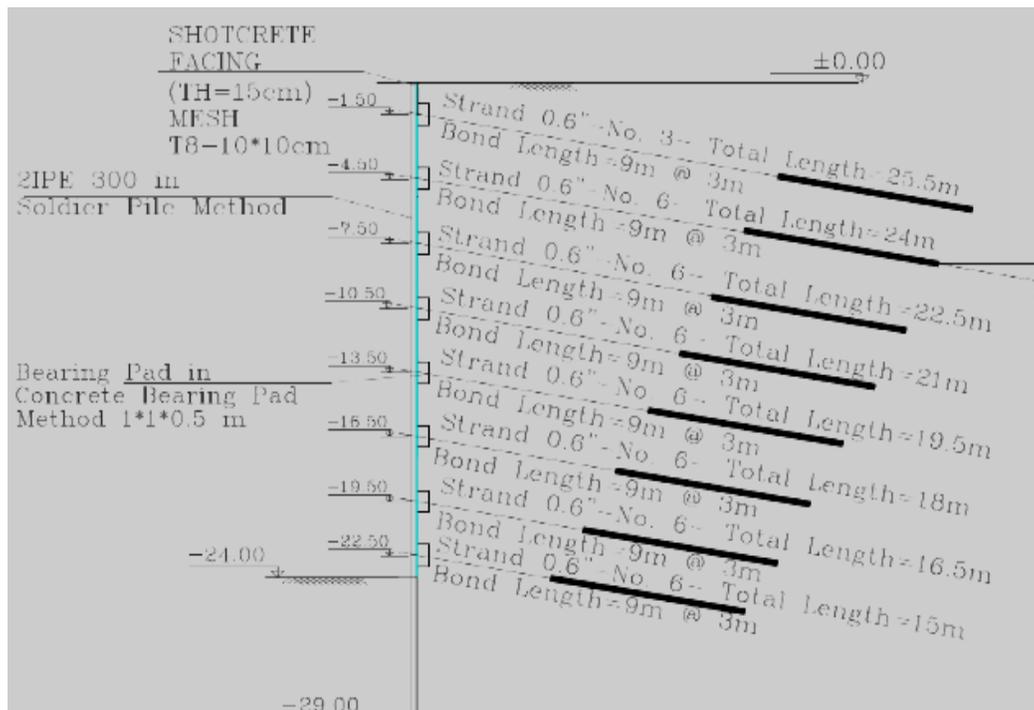


Figure 1. Details of nailing, anchorage and shell system [21]

Table 1. Details of retention system and excavation soil [21]

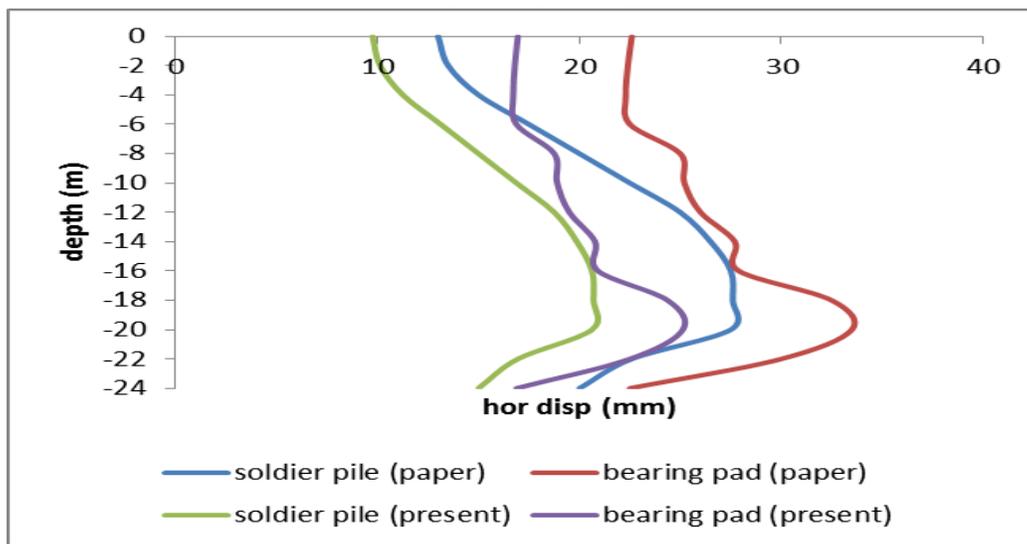
Soil type	γ (kN/m ³)	c (kPa)	ϕ (deg)	ψ (deg)	ν	E (MPa)
1	20	30	36	10	0.3	80

Parameter	γ (kN/m ³)	E (GPa)	Thickness (cm)	ν
Value	25	21	15	0.2

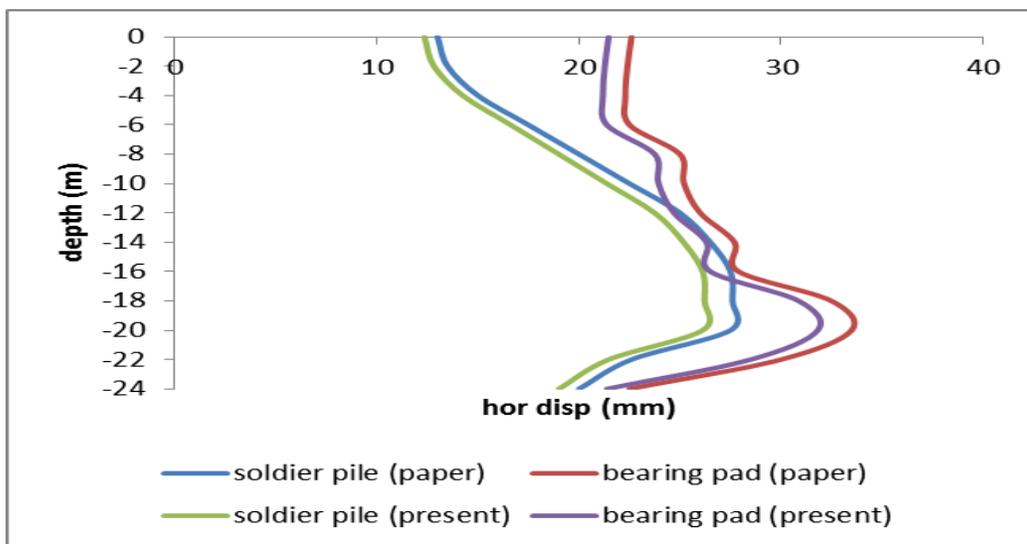
Element	Dimensions	γ (kN/m ³)	E (GPa)	ν	I (cm ⁴)
Pile	2IPE 300	78.5	210	0.3	1.67e4
Bearing pad	1×1×0.5 (m)	25	21	0.2	1.042e6

Figure 2 shows the comparison of the horizontal displacement of the excavation between the model of Rashidi et al. [21] and the modeling performed in this study. Based on the results, it was observed for the coarse meshing that the results of the authors' studies on the horizontal displacement were lower than for the paper results, but for the fine meshing, the results were the opposite. For the medium meshing, the results of the paper and the authors'

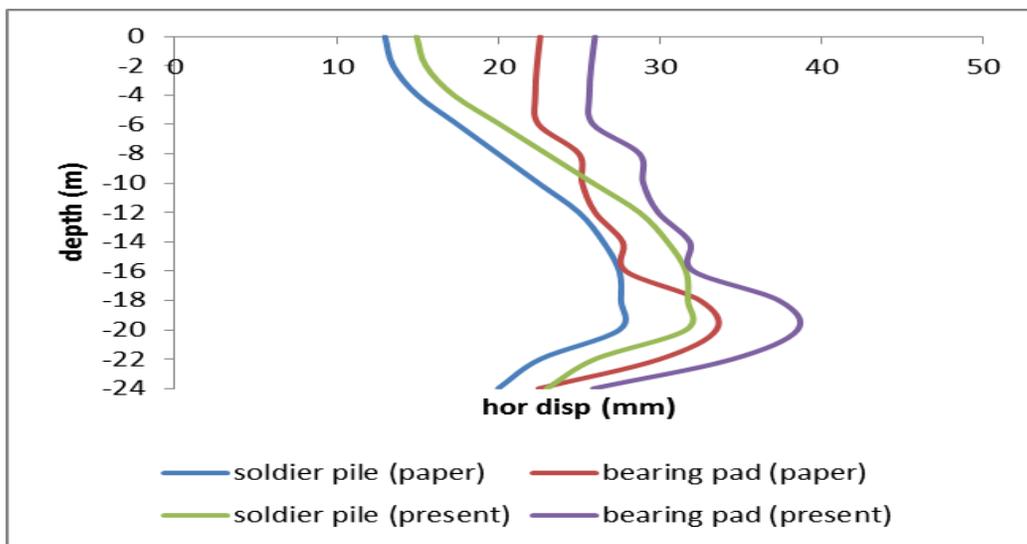
results were 5% different. Two points can be made in this section. First, the finite element method (FEM) results are sensitive to the meshing size, and second, the deformations increase and stresses decrease as the meshes become finer (so-called softer behavior models), and the deformations decrease and stresses increase as the meshes become coarser (so-called harder behavior model). Therefore, in any case, it is suggested to use the medium meshing.



a



b



c

Figure 2. Comparison of authors and paper modeling results (coarse meshing (a) - medium meshing (b) - fine meshing (c))

3. RESULTS AND DISCUSSION

Three different heights were considered to investigate the effect of height on the excavation response. [Figure 3](#) shows

the loading and support boundary conditions for the 3m high excavation.



Figure 3. Loading and support boundary conditions

The excavation is modeled in two-dimensional and plane-strain conditions, and the vertical distance of the anchors is assumed to be 0.5 m. The length of the anchors is assumed to be equal to the excavation height, and the inclination

angle is assumed to be 15°. [Figure 4](#) shows the results of the lateral displacement of shallow excavation for the 3m-high Mohr-Coulomb, Drucker-Prager, and Cam-Clay behavior models.

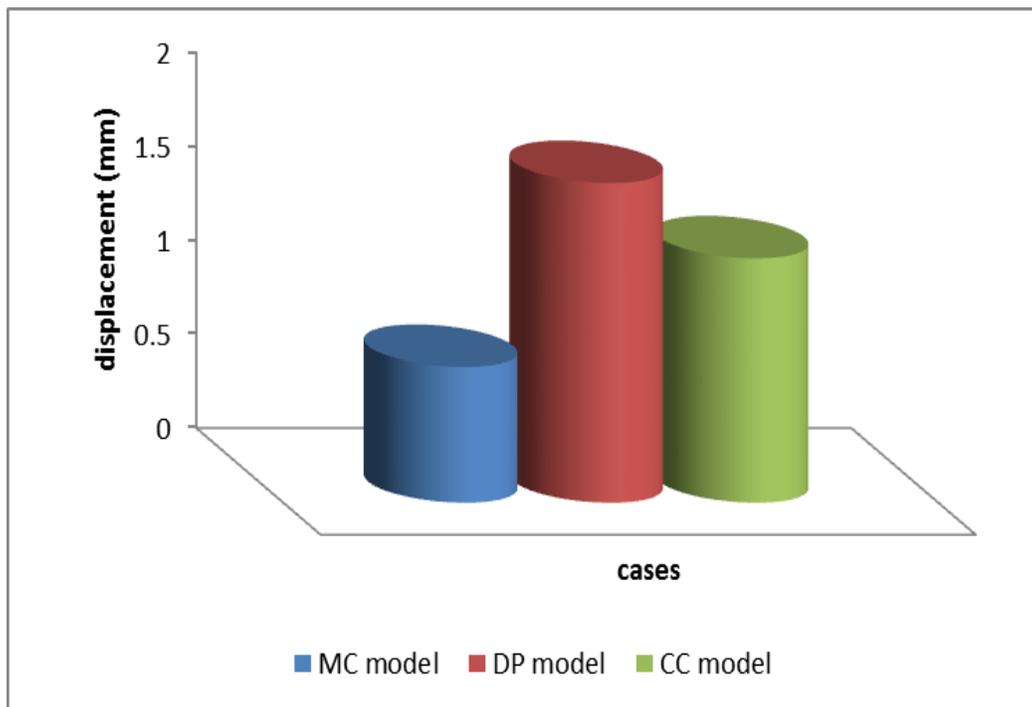


Figure 4. Comparison of lateral displacement of shallow excavation (3 m) for Mohr-Coulomb, Drucker-Prager and, Cam-Clay behavior models

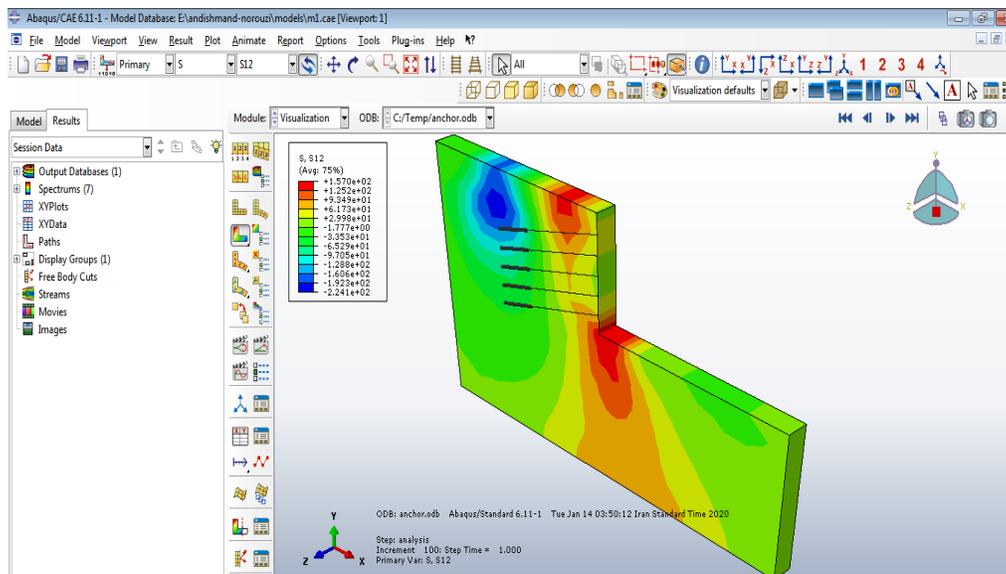


Figure 5. Shear stress distribution contours (Mohr-Coulomb model)

Figure 5 shows the shear stress distribution contours for the Mohr-Coulomb model. The results showed that the Mohr-Coulomb behavior model has the least displacement, and the Drucker-Prager behavior model has the largest lateral displacement for the excavation. The Drucker-Prager behavior model should be considered as a reliable criterion

for the design and control of the excavation because of the greater results regarding the lateral displacement of excavation and generally, excavation deformation. In general, it is suggested to use the Drucker-Prager behavior model in this case, which has the stress-dependent volume changes.

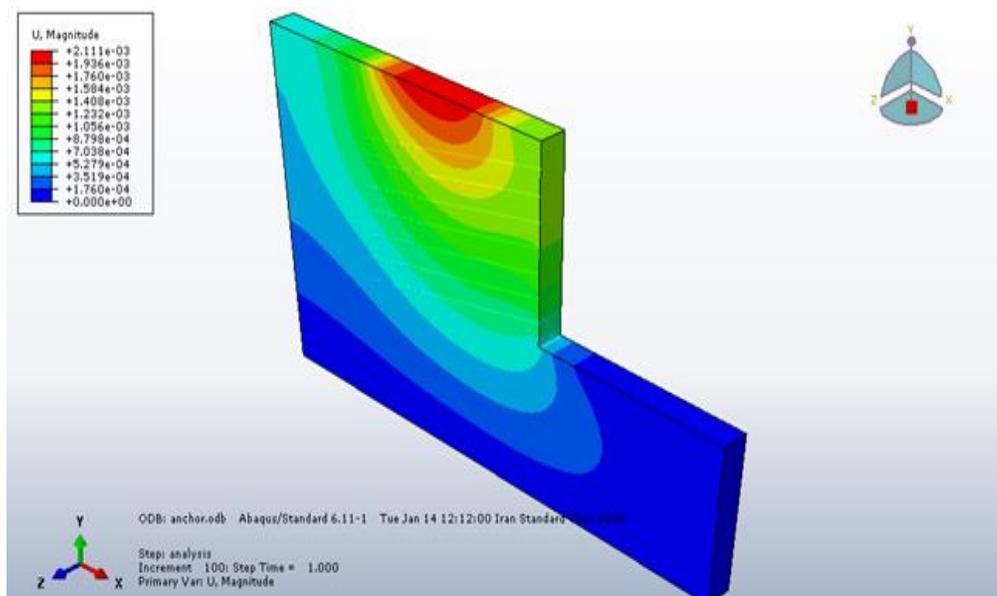


Figure 5. Deformation distribution contours (excavation depth: 8 m)

A model with a depth of 8 m was considered to evaluate the response of the excavation with the medium depth stabilized by the anchorage system based on different

behavior models. Figure 5 shows the deformation distribution contours for the model with a depth of 8 m.

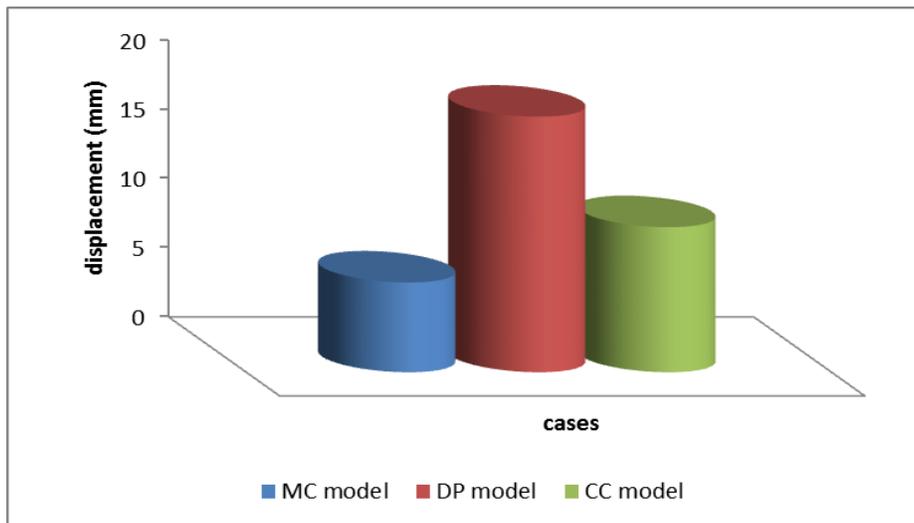


Figure 6. Comparison of lateral displacement of excavation with medium depth for Mohr-Coulomb, Drucker-Prager, and Cam-Clay behavior models

Figure 6 shows the lateral displacement of the excavation with medium depth for different behavior models in the 8m deep model. In this case, the Mohr-Coulomb behavior model had the least displacement, and the Drucker-Prager behavior model had the highest lateral displacement for the excavation. The Drucker-Prager behavior model should be considered as a reliable criterion for the design and control of the excavation because of the greater results regarding the lateral displacement of excavation and generally, excavation deformation. In general, it is suggested to use the Drucker-Prager behavior model in this case, which has the stress-dependent volume changes. Interestingly, the difference in the results for the lateral displacement of the excavation based on the Mohr-Coulomb and Drucker-

Prager behavior models was greater in this case (medium excavation) than the previous case (shallow excavation). Aghazadeh et al. and Yoo found in the study on the anchorage system that the Cam-Clay behavior model provides good results for the modeling with a medium depth [22,23]. A model with a depth of 14 m was also considered to evaluate the behavior of the deep excavation. The excavation was modeled in two-dimensional and plane-strain conditions, and the vertical distance of the anchors was assumed to be 1 m. The length of the anchors was assumed to be equal to the excavation height, and the inclination angle was assumed to be 15°.

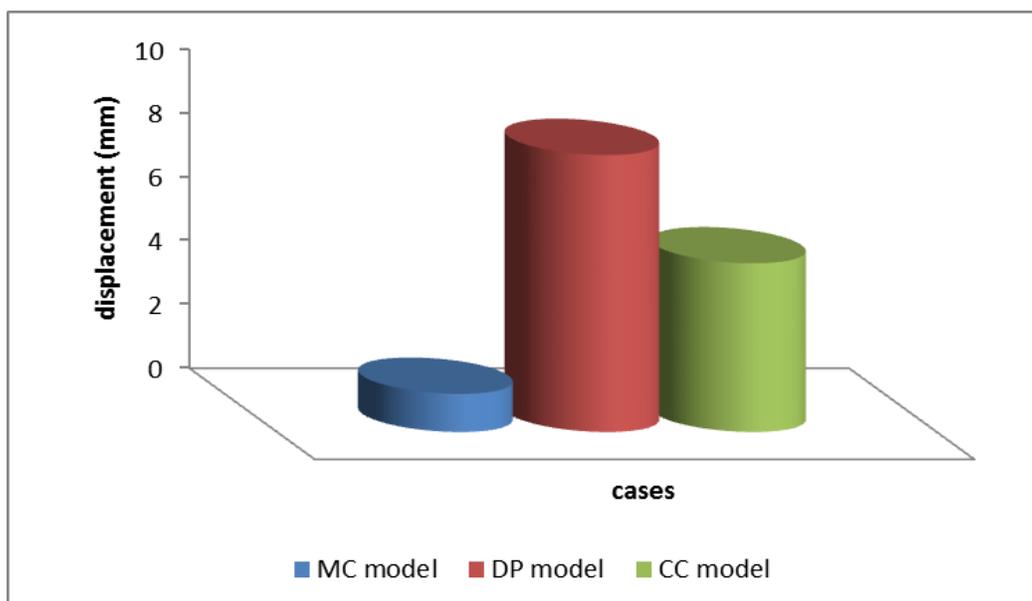
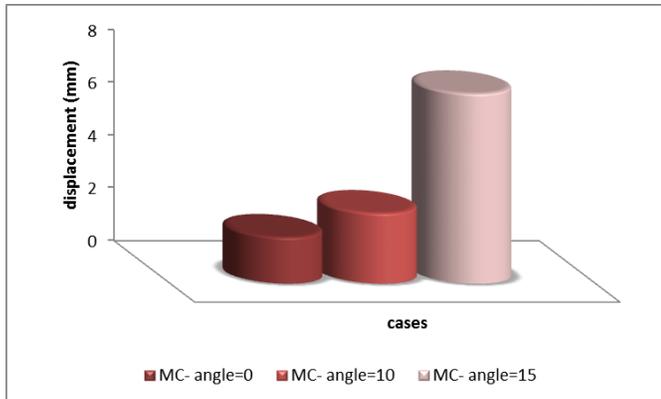


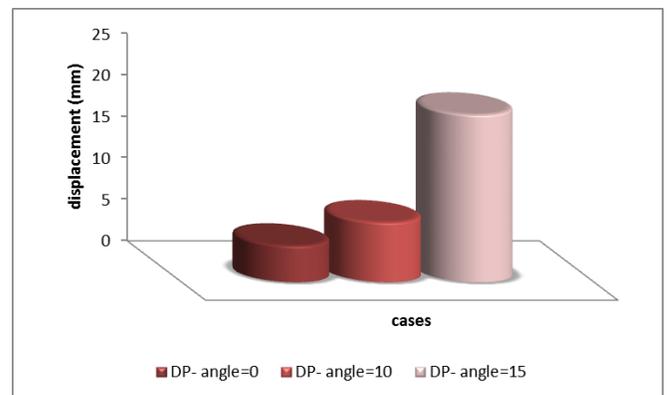
Figure 7. Comparison of lateral displacement of deep excavation (14 m) for Mohr-Coulomb, Drucker-Prager, and Cam-Clay behavior models

Figure 7 shows the lateral displacement of the excavation with a depth of 14 m in different behavior models. Before discussing the results of this section, it should be noted that the lateral displacement of the excavation, in this case, was less than that with the medium depth. This is because the excavation was deep, and a stronger anchor had to be selected (otherwise, the results of the excavation analysis were unstable). In this case, the Mohr-Coulomb behavior model had the least displacement, and the Drucker-Prager behavior model had the highest lateral displacement for the excavation. The Drucker-Prager behavior model should be considered as a reliable criterion for the design and control of the excavation because of the greater results regarding

the lateral displacement of excavation and generally, excavation deformation. In general, it is suggested to use the Drucker-Prager behavior model in this case, which has the stress-dependent volume changes. Interestingly, the difference in the results for the lateral displacement of the excavation based on the Mohr-Coulomb and Drucker-Prager behavior models was significantly greater in this case (deep excavation) than the previous case (medium excavation). The results of the impact of depth on the behavior of the excavation reinforced with the anchorage system have a good agreement with the results of Saeedi et al. [24].



a. Mohr-Coulomb

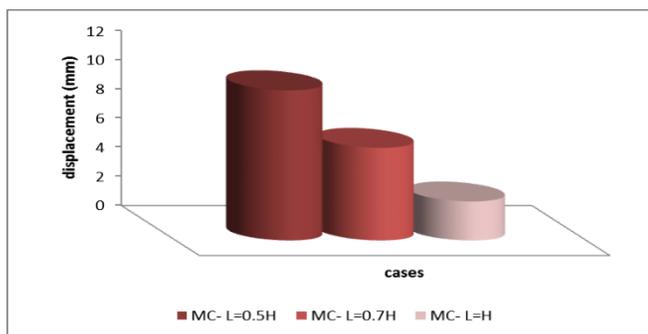


b. Drucker-Prager

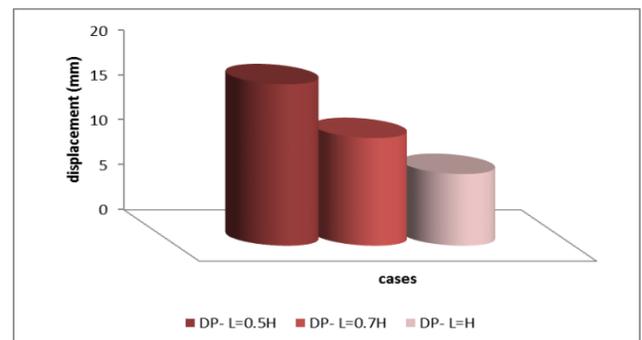
Figure 8. Comparison of lateral displacement of excavation for Drucker-Prager and Mohr-Coulomb behavior models for different anchor angles of 0, 10, and 15°.

To investigating the effect of anchor angle on the excavation response, the anchor inclination angles of 0, 10, and 15° were considered, and the results were evaluated and compared for different behavior models. To compare the results, only two Mohr-Coulomb and Drucker-Prager behavior models were considered. As shown in Figure 8, it

is evident that increasing the anchor angle results in the increase in the lateral displacement of the point above the excavation, and in addition, in all cases, the lateral displacement results for the Drucker-Prager behavior model are greater than the similar results based on the Mohr-Coulomb behavior model.



a. Mohr-Coulomb



b. Drucker-Prager

Figure 9. Comparison of lateral displacement of excavation for Drucker-Prager and Mohr-Coulomb behavior models for anchor length equal to 0.5, 0.7, and 1 times excavation height

In this study, to investigate the effect of anchor length on the excavation response, the anchor lengths equal to 0.5,

0.7, and 1 times the excavation height were evaluated and compared for different behavior models. For the

comparison, two Mohr-Coulomb and Drucker-Prager behavior models were considered, and the nail angle was assumed to be 10° . As can be seen in [Figure 9](#), as the anchor length increased in each case, the lateral displacement of the

point above the excavation decreased, and this decrease was significant in each case. The lateral displacement was also greater in the case of using the Drucker-Prager model.

4. CONCLUSION

In this study, the effect of soil behavior model on the drilling response of the excavation stabilized by anchorage system was evaluated in the parametric studies using the software, finite element numerical method, and ABAQUS software and the following results were obtained:- Based on the results for the shallow excavation, it was found that the Mohr-Coulomb behavior model has the least displacement, and the Drucker-Prager behavior model has the greatest lateral displacement. The Drucker-Prager behavior model should be considered as a reliable criterion for the design and control of the excavation because of the greater results regarding the lateral displacement of excavation and generally, excavation deformation. In general, it is suggested to use the Drucker-Prager behavior model in this case, which has the stress-dependent volume changes.- For the excavation with medium depth, the Mohr-Coulomb behavior model had the least displacement, and the Drucker-Prager behavior model had the greatest lateral displacement. The Drucker-Prager behavior model should be considered as a reliable criterion for the design and control of the excavation because of the greater results regarding the lateral displacement of excavation and

generally, excavation deformation.- For the deep excavation, the Mohr-Coulomb behavior model had the least displacement, and the Drucker-Prager behavior model had the greatest lateral displacement for the excavation. The Drucker-Prager behavior model should be considered as a reliable criterion for the design and control of the excavation because of the greater results regarding the lateral displacement of excavation and generally, excavation deformation.

- It was found that increasing the anchor angle results in the increase in the lateral displacement of the point above the excavation, and in addition, in all cases, the lateral displacement results for the Drucker-Prager behavior model are greater than the similar results based on the Mohr-Coulomb behavior model.

- It was found that as the anchor length increased in each case, the lateral displacement of the point above the excavation decreased, and this decrease was significant in each case. The lateral displacement was also greater in the case of using the Drucker-Prager model.

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AUTHORS CONTRIBUTION

This work was carried out in collaboration among all authors.

CONFLICT OF INTEREST

The author (s) declared no potential conflicts of interests with respect to the authorship and/or publication of this paper.

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