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Examining the Four Parameters of Genetic Algorithm in Order to Obtain the Best Solution for Transportation Network Design Problems

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ABSTRACT

Usually, after carrying out network design studies, the question arises of how the output variables are affected by the input variables of a model. In other words, how can one use a method to change the inputs of a statistical model in an organized manner so that the effects of these changes can be predicted on the output of the model. This work is usually done by sensitivity analysis, so in this sensitivity analysis study, two parameters, the optimality degree of the objective function of the problem and the time to solve the problem, are examined in relation to the change in the four parameters of the genetic algorithm. In other words, the purpose of this study is to find the best combination of genetic algorithm parameters (survival probability, mutation rate, recombination probability, population) and also to check the changes of the mentioned criteria against the four parameters of the problem, so that the best solution is obtained. The results showed that increasing the initial population will improve the answer. This is because a more accurate search is performed with a larger number of solving factors in the feasible space. The solution time of the model also shows the same 65% as the optimality of the search objective function following the best reproduction probability value. The higher the survival probability, the more chromosomes of the current generation will be transferred to the next generation without any operation, which will naturally reduce the solution time.

Keywords: Genetic Algorithm, Network Design, Survival Probability, Mutation Rate, Recombination Probability, Population.

1. INTRODUCTION

Network design problem is considered one of the most basic theoretical issues in the field of urban transportation planning, the subject of which is the economic evaluation of urban transportation network development in order to meet future travel demand. Solving the problem of network design will answer the question of how to develop the capacity of the existing links in the network or build a new link and improve the network in general so that the objective function operates at the highest possible level of efficiency and the various limitations caused by various aspects of the matter. Such as resource limitations and specific limitations of the assumed objective function, should not be violated [1,2], Network design problem has been investigated by different deterministic, heuristic and meta-heuristic solution methods. The first attempts to use meta-innovative methods were made using the heating and cooling method [3, 4] Also, cases of using genetic algorithm [5,6] also developed over time. Genetic algorithm methods can give land use policymakers the ability to evaluate [7] , and also have the ability to calibrate parameter values [8,9] and also improve the accuracy of the model [10].

To get familiar with genetic algorithms, imagine that an optimization problem is to be solved with genetic algorithms, for this purpose, each solution to the problem must first be shown with a chromosome that is a combination of different genes, in fact, each gene represents a trait or characteristic. It is from the answer [11]. Then, like the process of evolution, an initial set of chromosomes is produced randomly (primary population). Then, a very simple mathematical routine is used and a number of the best answers (chromosomes) are separated from the others, then, in the same way that living organisms reproduce in nature, the selected chromosomes are mixed with each other and new chromosomes are produced from their combination (children) so that these chromosomes acquire their characteristics from the parents and in the same way the second and third generation and the best genes are separated and the rest are removed and in the same way the process of selection, combination and mutation continues until reaching the right answer [12]. In the stages of genetic algorithm, the first step is to create the initial population. The initial population consists of a series of problem answers that are randomly generated [13]. The second important point in creating an initial population is its size. The size of the initial population has a significant effect both on the execution time of the algorithm and on the quality of the algorithm's answer, the size of the initial population depends on the type of problem and its characteristics [14]. Based on the experiences gained from the application of genetic algorithms in solving various optimization and practical problems, it has been concluded that the population size is 100. But there is no guarantee that it is appropriate, because there is no procedure or instruction to obtain the optimal value of the population size, and usually based on experience and testing different values, as well as checking the performance of the algorithm, the size of the initial population can be roughly determined [15].

Just as in the process of evolution of nature (production like living organisms), a mutation is created in the production of children by randomly changing the genes of the parents' chromosomes. Here too, by randomly changing one or more genes of the chromosome, a mutation is created in the answer population, and the search may move from one area of the answer space to another [16]. During the execution of the steps of the algorithm, the algorithm may search for a suitable solution in a region of the solution space, and if the mutation operator is not applied at different times. The algorithm leads to providing a suitable local solution, which is not a good solution at all, so to avoid this, it is necessary to apply the mutation operator every once in a while in the algorithm. Its operator is usually applied to a parent and produces a child [17]. The basis of the work of the genetic algorithm for solving network design and network pricing problems is based on the selection of an initial population of genes [18]. This initial population then produces new generations based on the rules governing the genetic algorithm, i.e., reproduction, intersection and mutation, and each generation must be evaluated in order to obtain the optimal solution. Each population of genes is, in fact, a set of answers that determines which links are improved and to what extent, and to what extent and on which links are affected [19] One of the studies conducted in relation to the problem of network design is the study conducted by Zhou, Wei and Hu in 2009. In this study, Zhou and his colleagues try to compare the results of solving the network design problem, using two meta-heuristic algorithms of genetics and simulated heating and cooling of metals. In general, the authors of the article have concluded that the speed of solving the problem by genetic algorithm is much higher than the other method and it gives much better results in low traffic demand volumes, however, they showed that the amount of demand in the network increases The results of two algorithms approach each other, but the genetic algorithm still has a better solution time [20]. Table (1) shows a number of studies conducted in the field of network design using the genetic algorithm solution method.

Table 1. Summary of literature review

Researcher	Year	Objective function	Limitations	Solution method
Liu et al.	2020	Maximizing the public interest of network users.	User balance traffic allocation constraints, budget constraints, toll rate constraints.	Binary Genetics Cutting Network Search Network
Safirova et al.	2007	Maximize the net benefits of the network.	User balance traffic allocation constraints, budget constraints, toll rate constraints.	Genetic algorithm
Sumalee	2004	Maximizing the net benefits of the network with social justice in mind.	User balance traffic allocation constraints, budget constraints, toll rate constraints, social justice constraints.	Modified genetic algorithm called branch and edge algorithm
Soto et al.	2023	Maximizing the public interest of network users.	Traffic allocation restrictions, user balance, budget restrictions, toll rate restrictions, land use restrictions and conditions.	Genetic algorithm
Zheng et al.	2022	Maximize social welfare.	Traffic allocation restrictions, user balance, budget restrictions, toll rate restrictions, land use restrictions and conditions.	Genetic algorithm
Ardila-Gomez et al.	2021	Maximize social welfare.	Traffic allocation restrictions, user balance, budget restrictions, toll rate restrictions, land use restrictions and conditions.	Genetic algorithm

2. METHODOLOGY

2.1. Case Study

In this section, a network with medium dimensions, twenty-nine nodes, fifty-one non-directed links and twelve origins and destinations has been selected. Since this network is a part of an urban network, all links are in the position of urban roads where the flow is maintained in both directions. This network is related to the city of Benevento in Italy.

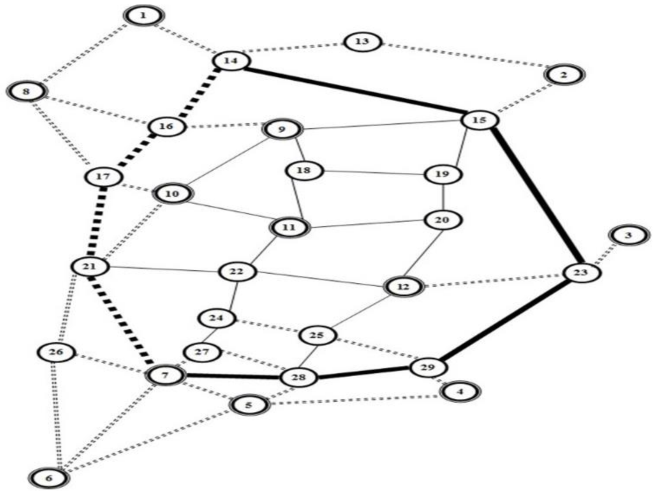

Figure 1. The hypothetical network under study

In the following, a model will be presented that can be used to determine the optimal amount of tolls and the capacity of network links on a predetermined toll collection belt. The proposed model is as follows:

High level problem:

$$Max\ TCS = \sum_{rs}\left[\int_0^{q^{rs}} D_{rs}^{-1}(v)dv - \sum_p f_p^{rs}\eta_p^{rs}\right] \quad (1)$$

$$= \sum_{a\epsilon A/B}\psi t_a \delta_a^p \eta_p^{rs} + \sum_{b\epsilon B}(\psi + \rho_b)\delta_a^p t_b \qquad \forall r,s,p \quad (2)$$

$$IC \leq B + TT \quad (3)$$

$$TIC = \sum_a\left(\frac{y_a}{1000}\gamma_0\frac{t_a^0}{s_0}\right) \quad (4)$$

$$TT = \sum_{b\epsilon B}(\rho_b V_b) \quad (5)$$

$$y_a \geq 0 \quad (6)$$

$$0 \leq \rho_b \leq \rho_{b,Max} \quad (7)$$

Low level problem:

$$Min \sum_a \int_0^{V_a} t_a(w)dw - \sum_{rs}\int_0^{q^{rs}} D_{rs}^{-1}(w)dw \quad (8)$$

$$= \sum_a t_a \delta_a^p u_p^{rs} \quad (9)$$

$$= \sum_{rs}\sum_a f_p^{rs}\delta_a^p V_a \quad (10)$$

$$= \sum_P f_p^{rs} \ q^{rs} \quad (11)$$

$$f_p^{rs} \geq 0 \quad (12)$$

$$q^{rs} \geq 0 \quad (13)$$

$$t_a = t_a^0\left(1 + \alpha\left(\frac{V_a}{(c_a + y_a)}\right)^{\beta}\right) \quad (14)$$

The parameters of the above relationships are as follows:

V_a is flow volume on link a

t_a is travel time of link a

q^{rs} is Travel demand between origin and destination, r and s

$D_{rs}^{-1}(.)$ is reverse of the demand function

u_p^{rs} And u^{*rs} are Travel time of origin-destination, r and s, on route p and minimum travel time between r and s, respectively

δ_a^p is an index that is equal to 1 if link a is a part of route p; otherwise, it is zero

f_p^{rs} is flow volume on route p between origin and destination, r and s

t_α^0 is free-flow travel time of link a

C_a is initial capacity of link a

y_a is increased capacity of link a

α and β are constant values of travel time function by Federal Highway Administration

η_p^{rs} is generalised travel cost from origin r to destination s on route p

ψ is the unit value of travel time

A and B are total set of network links and the links with toll, respectively, while $B \subset A$

ρ_b is received toll on link b

$\rho_{b,max}$ is maximum permitted amount of toll on link b

TT is total income from toll collection in the network

TIC is the total costs of network development and improvement

TB is total available budget

y_0 is the construction cost of a motion line

S_0 is free-flow travel speed on the link

TCS is total social benefits of network pricing and development plan

γ_0 is cost of adding a motion line

In the study of Bigdeli Rad, the network design problem is expressed as a two-level optimization problem. The goal of the high-level problem is to minimize the total network travel time plus the investment cost in order to increase the capacity of the current network links. The low-level problem also includes solving the traffic allocation problem with constant demand. Frank-Wolf algorithm is used to solve the low-level problem. Then, by using the answers obtained from this stage and by using the genetic algorithm, the final answers will be obtained in the general state, in which the amount of optimization of the links of the network and the amount of optimal tolls that can be received in the links of the toll collection belt are predetermined.

3. RESULTS AND DISCUSSION

In this part, according to the introduced network, and the results of this table are obtained based on the sensitivity analysis of two parameters, the degree of optimality of the objective function of the problem and the time of solving the problem with respect to the change in the four parameters of the genetic algorithm. In other words, at this stage, the primary goal is to find the best combination of genetic algorithm parameters, so that the best solution is obtained. The changes of the mentioned criteria against the four parameters of the problem are shown in graphs (2) to (9). As seen in table (2). Solving the problem with a population of 100 had the best solution, and the genetic algorithm reached the optimal solution after 788 generations, which lasted for 10973 seconds. In other words, each repetition of the genetic algorithm in simultaneously solving the problems of network design and network pricing on the experimental network has lasted only 13.92 seconds, considering that the presented model is a two-level problem and its solution every time By the genetic algorithm, the problem of traffic allocation by the Frank-Wolf method and the problem of finding the shortest path between all the pairs of origin-destinations are solved by the Dijkstra method every time, the obtained solution time is considered acceptable.

Table 2. Best results

<table>
<tr><th colspan="5">Results of problem solving by GRG method</th></tr>
<tr><td>solution time</td><td colspan="2">Optimal toll rate</td><td>Average travel time of each car</td><td>The objective function</td></tr>
<tr><td>81678 seconds</td><td colspan="2">1.82 currency</td><td>604 seconds</td><td>4.09*10^9</td></tr>
<tr><th colspan="5">The results of problem solving by genetic algorithm method</th></tr>
<tr><td>solution time</td><td>Number of generations</td><td>Optimal toll rate</td><td>Average travel time of each car</td><td>The objective function</td></tr>
<tr><td>10973 seconds</td><td>788</td><td>1.85 currency</td><td>592 seconds</td><td>4.16*10^9</td></tr>
<tr><th colspan="5">The best values of genetic algorithm parameters</th></tr>
<tr><td>Probability of surviving</td><td colspan="2">Mutation rate</td><td>Possibility of recombination</td><td>population</td></tr>
<tr><td>0.15</td><td colspan="2">0.35</td><td>0.65</td><td>100</td></tr>
</table>

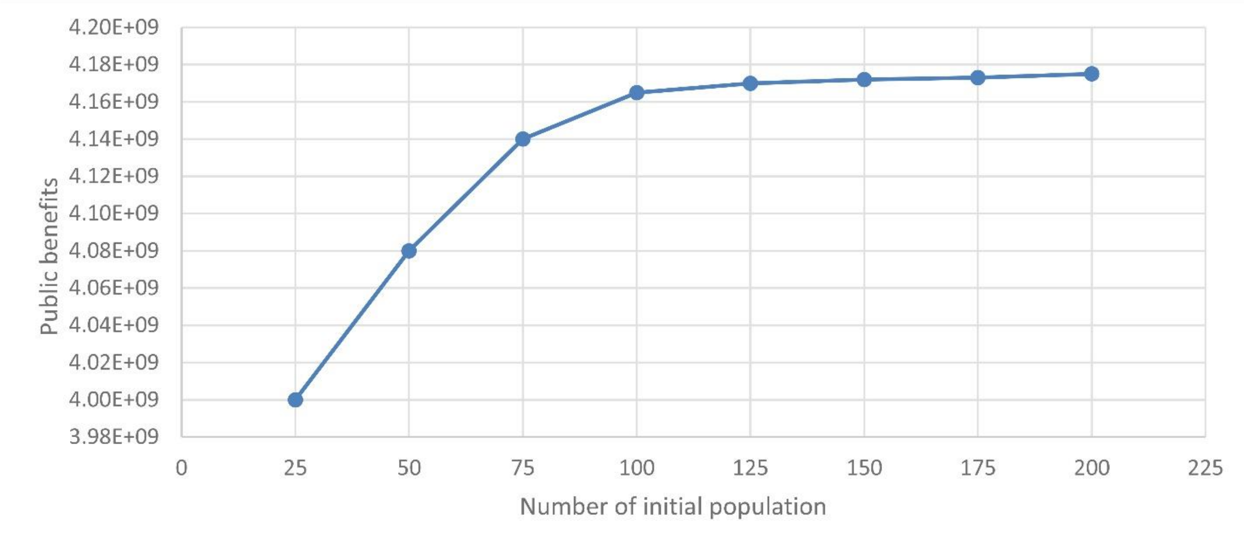

Figure 2. Analysis of the sensitivity of the objective function of the model to the population size parameter in the genetic algorithm

Figure (2) shows the sensitivity analysis of the objective function of the problem to the number of the initial population in the genetic algorithm. This analysis was done in the initial population range between 25 and 200 chromosomes. In fact, the initial population expresses the number of chromosomes that perform the search process in the feasible space by the genetic algorithm method and through three genetic behaviors: recombination, mutation, and retention. It can be seen that increasing the initial population will improve the answer. This is because a more detailed search is performed with a larger number of solving factors in the feasible space. According to the results of this graph, the improvement of the problem solution and the optimal level of the objective function has a relatively steep slope up to the number of 100 chromosomes, and from that point onwards, the slope of the graph is planted. Especially from the number of 150 chromosomes onwards, the improvement in the optimality of the objective function has become extremely minor and has not had a significant growth. The next graph, i.e. Figure (3), shows the analysis of problem solving time per second in relation to the initial population in the genetic algorithm.

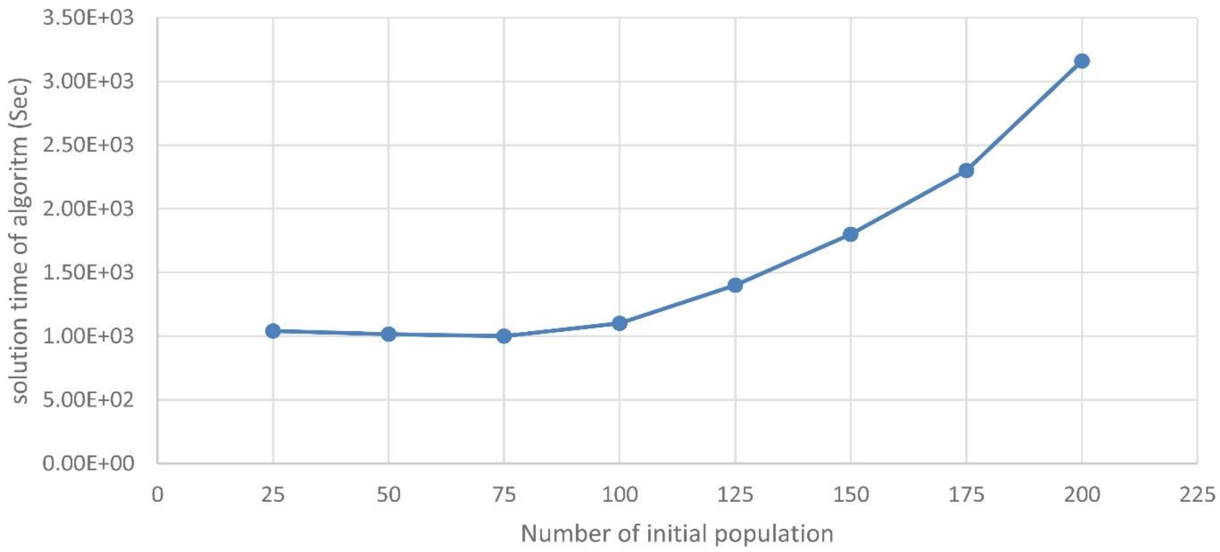

Figure 3. Sensitivity analysis of model solution time to population size parameter in genetic algorithm

In figure (3), this time a sensitivity analysis has been done from the problem solving time to the number of the initial population in the genetic algorithm. This analysis was also performed in the initial population range between 25 and 200 chromosomes. In examining the effect of the initial population on the speed of solving the model by the genetic algorithm, two points should be noted. First, the fact that the larger the size of the initial population used in solving the problem, the time to solve the model for one iteration of the algorithm increases because the solution factors increase and the scope of the algorithm implementation increases. On the other hand, it is expected that with a larger number of the initial population, each iteration of the algorithm will perform a more accurate search for the objective function, and as a result, a smaller number of generations will be needed to solve the problem. Therefore, increasing the number of the initial population at the same time can have a positive or negative effect on solving the problem. This fact can be seen in graph (3). As can be seen, the minimum of the curve is at the number of 75 chromosomes as the initial population, and from that point on, the increasing effect of the solution time due to the number of the initial population in contrast to the decreasing effect of the solution time due to the reduction of the number of generations in the search for the solution has faded. And the time to solve the model has increased significantly, especially from the number of 125 chromosomes as the initial population.

Two curves (2) and (3) should be checked side by side. that the simultaneous examination of two curves shows that the number of 100 generations is the optimal value that keeps the solution improvement time and the optimality of the objective function of the problem solving time at an acceptable level, and in fact the best number for the initial chromosome number considering both the solving time and Optimality is the objective function.

Recombination probability is one of the parameters of the genetic algorithm that determines how many percent of the population in each generation should mix together to create the next generation. This number can naturally vary from zero to one hundred. As explained before, a small value for this probability will cause a large number of chromosomes of the current generation to make their way to the next generation, which will reduce the probability of improving the solution in the next generation. On the other hand, this analysis of a large value of this parameter causes a greater number of chromosomes of the current generation to undergo changes in the movement towards the next generation. This issue will cause the good answers obtained in the current generation to be lost in the movement towards the next generation. As a result, the elitist feature of the genetic algorithm becomes ineffective. This explanation shows the importance of finding the best value for the probability of reproduction. Of course, this possibility and the other two possibilities, mutation and survival, are better to be optimized at the same time, in which case, even by limiting the range of changes of these three possibilities, the search range is greatly expanded and the time to solve the sensitivity analysis increases greatly, on the other hand, there is a chance of finding a better answer. From the mode of independent sensitivity analysis, one possibility does not increase much, as a result, this sensitivity analysis has been done for the above-mentioned three possibilities separately. Diagram (4) shows the sensitivity analysis of the objective function of the problem in relation to the probability of recombination in the genetic algorithm process.

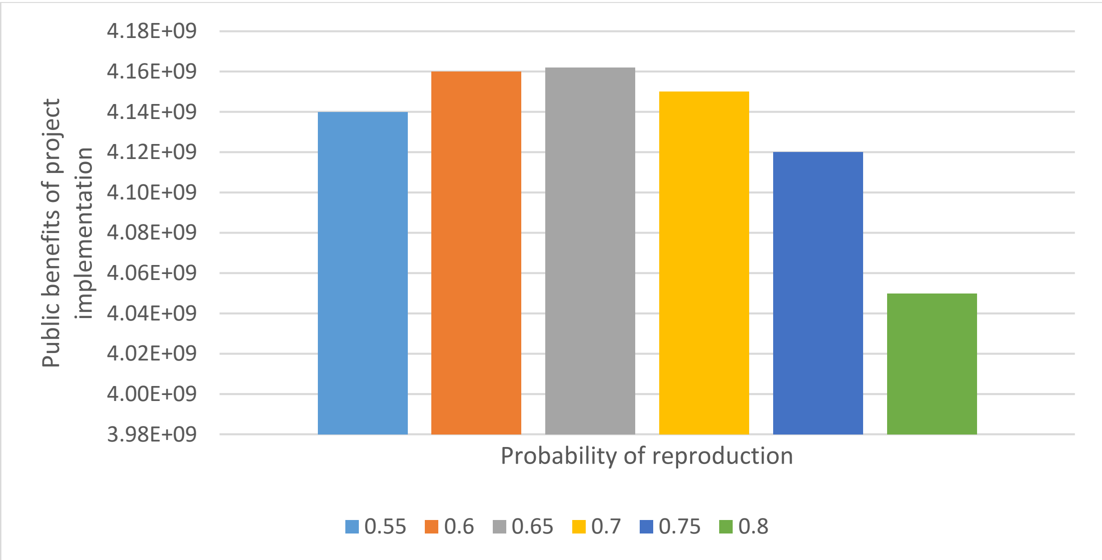

Figure 4. Analysis of the sensitivity of the objective function of the model to the reproduction probability parameter in the genetic algorithm

The sensitivity analysis of the reproducibility parameter is done in the probability range of 55 to 80 percent. The selection of this interval has been done experimentally and by repeatedly performing the sensitivity analysis process. Figure (5) also shows the sensitivity analysis of problem solving time against the reproduction probability parameter. As can be seen in both graphs, the shape of both graphs is bowl-shaped, which happened at the time of solving the minimum of the curve and at the optimality of the objective function of the maximum of the curve in the probability of reproduction of 65%. This problem indicates the fact that selecting 65% of chromosomes in each generation to perform reproduction operations to move to the next generation has reduced both the optimality of the objective function of the problem and the time of solving the problem. Regarding the optimality of the objective function, a range of 108 units of money has been obtained for different values of the probability of reproduction, which is about 2% of the optimal objective function. In other words, choosing different values for the parameter of the probability of reproduction of only 2% can be effective in the amount of public benefits of the simultaneous design and pricing of the network. Of course, considering that the issue of network design and pricing is basically an optimization model, even 2% optimization cannot be ignored and sensitivity analysis is necessary to achieve it.

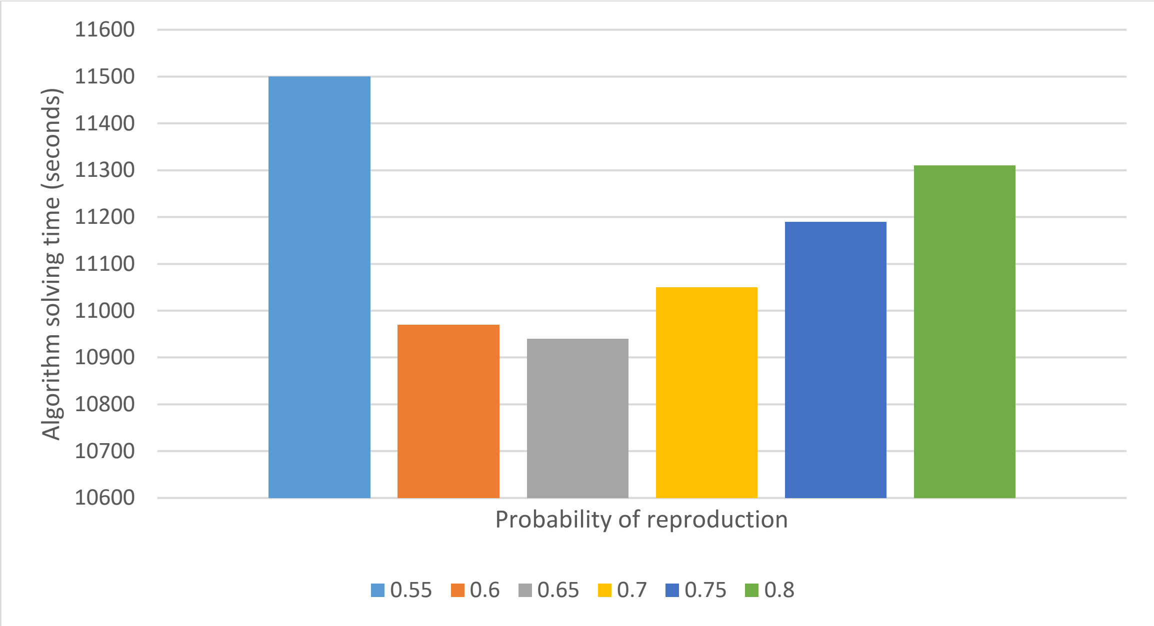

Figure 5. Analysis of the sensitivity of model solution time to the reproduction probability parameter in the genetic algorithm

The solution time of the model, like the optimality of the search objective function, shows the same 65% reproduction probability after the best value. Here, the range of solution time changes for different values of the reproduction function is a little more than 200 seconds, which is still about two percent compared to the optimal solution time of 10973 seconds. Although in a time period of around three hours, two percent or in other words around three minutes seems to be an insignificant amount, but in solving the problems of network design and pricing, the goal is usually to define a suitable framework to generalize to problems with real dimensions and in Such issues, which will take several days to solve, a two percent reduction in solving time will be significant in terms of cost.

Figures (6) and (7) respectively show the sensitivity analysis of the two optimality criteria of the objective function and the solution time against the third parameter of the genetic algorithm, i.e. mutation probability. Mutation is one of the three operators of the genetic algorithm and it expresses the percentage of chromosomes of the current generation that are not combined and reproduced, but changed by the rule of mutation and transferred to the next generation. This feature of the genetic algorithm is very important and it is very necessary to get the best value of it, because based on this feature, the genetic algorithm will be able to search different areas of the possible space for the optimal solution of the problem and avoid stopping and convergence. Around local optimal solutions to be saved to a large extent.

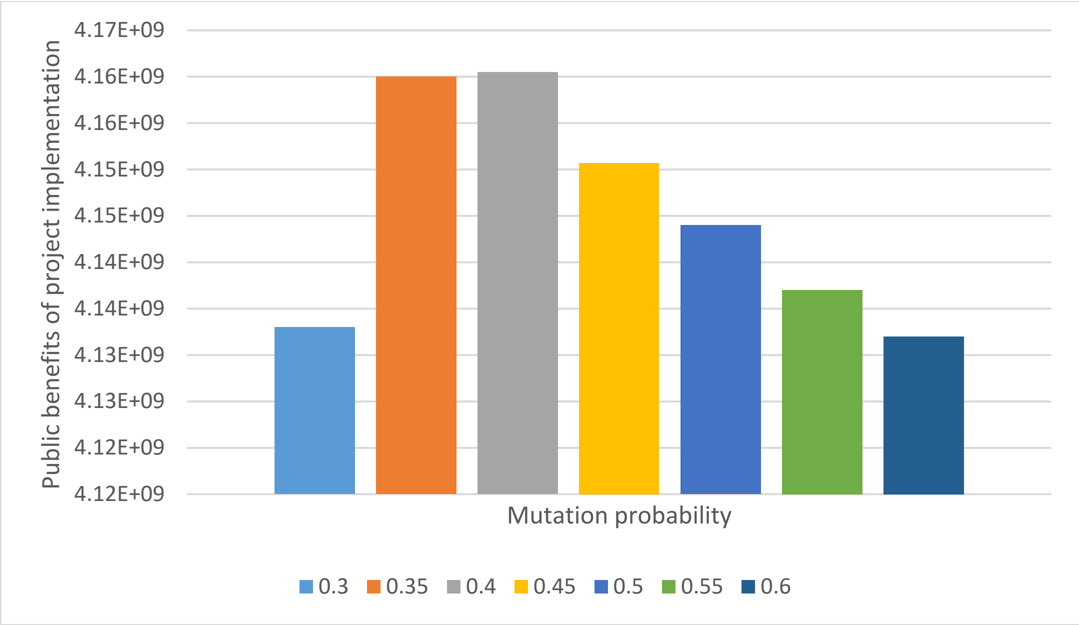

Figure 6. Analysis of the sensitivity of the objective function of the model to the reproduction probability parameter in the genetic algorithm

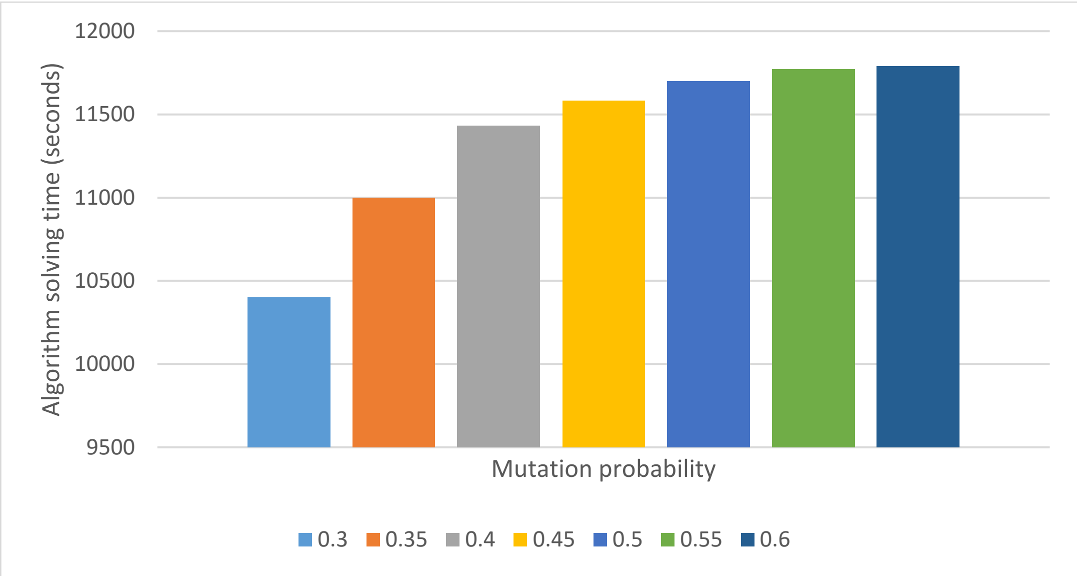

Figure 7. Analysis of the sensitivity of model solution time to the reproduction probability parameter in the genetic algorithm

According to Figure (6), the best solution to the problem is obtained for a 40% mutation probability, but the time to solve the problem with this mutation probability is significantly more than the time to solve the problem with a 35% mutation probability, according to the results of Figure (7). A closer look at the results shows that while the change in the optimal solution of the problem in the movement from the 35% jump probability to the 40% jump probability is less than 0.003%, the problem solving time increases by more than 4% in this change of probability. It was stated before that the main goal of pricing and network design is to develop a suitable framework to solve problems and large models with real dimensions. With this explanation, in real problems and even in this network of medium-sized studies in this research, the comparison of benefit to cost shows that an increase of more than 4% in the solution time has a serious effect on increasing the costs of the management department in the stage of problem solving and decision-making, and therefore more It is to be ignored. On the other hand, the change in the optimality of the objective function is a very small value that even in large dimensions cannot have a significant effect on the model results and change the basic decisions in pricing or network design for the project horizon. With this explanation, the mutation rate of 35% is selected for the final solution of the model by genetic algorithm.

Finally, the last parameter of the genetic algorithm is the survival probability. The survival probability determines the percentage of elite chromosomes that provide the best solution to the problem and must be passed on to the next generation without change. This feature of the genetic algorithm guarantees that if it finds a good answer, the algorithm will not abandon it under the effect of its evolutionary nature and will continue to search around it for better answers. With this explanation, it is clear that the greater the probability, the more number of chromosomes of the current elite generation will be recognized and will be transferred to the next generation in the process of solving the problem in the genetic algorithm. Two Figures (8) and (9) show the optimality and the solution of the objective function of the model and the problem solving time for the changes of the survival probability parameter in the range of 0.05 to 0.25.

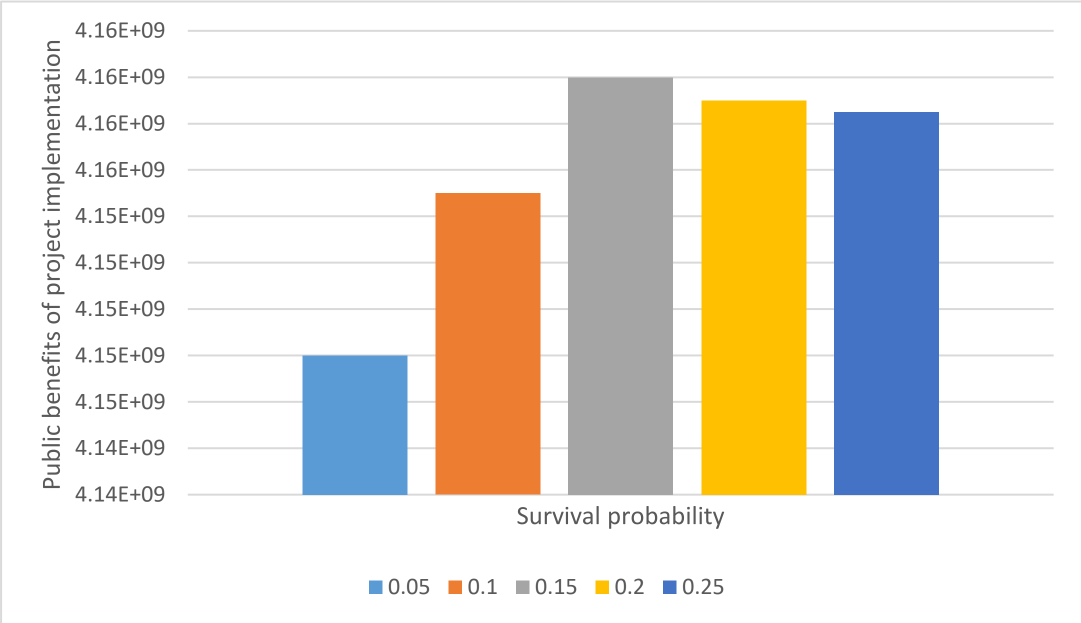

Figure 8. Analysis of the sensitivity of the objective function of the model to the probability parameter of the genetic algorithm

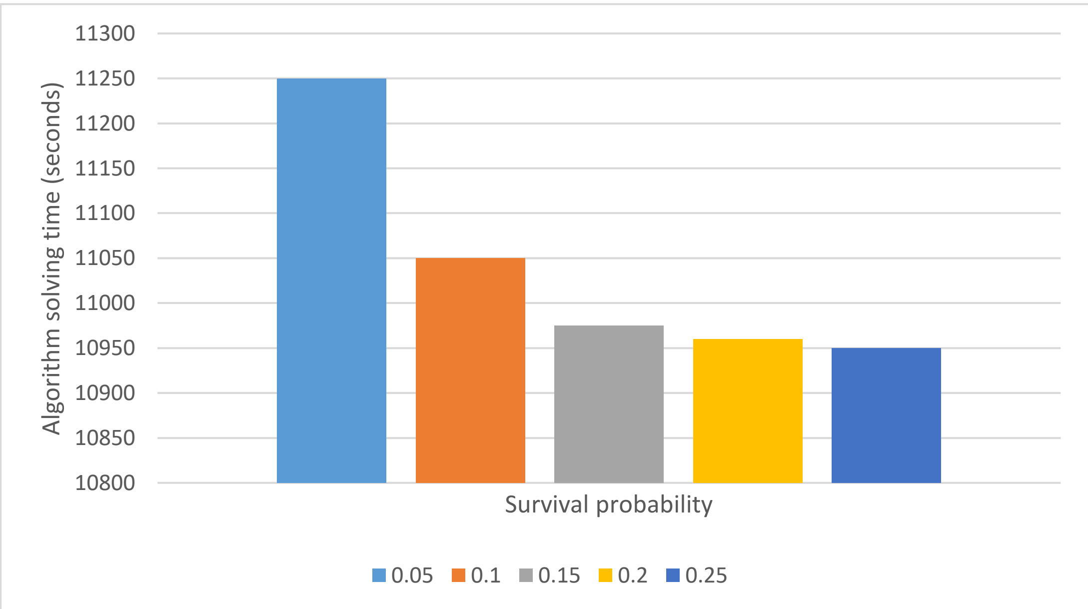

Figure 9. Analysis of the sensitivity of model solution time to the reproduction probability parameter in the genetic algorithm

Figure (8) indicates that the trend of changes in the optimality of the objective function of the problem versus the changes in the probability of stopping in the selected interval has a parabolic shape, the peak of which is placed in the probability of stopping at 15%. The difference between this point and our previous point, i.e. the probability of remaining open is 10%, is about 0.1% improvement in the optimality of the objective function of the problem, and the solution time has also decreased. Regarding the reduction of the solution time, it should be noted that the higher the survival probability, the more number of chromosomes of the current generation will be transferred to the next generation without any operation, which will naturally reduce the solution time. But graph (9) shows that with the increase of this probability, the severity of its reduction effect on the time of solving the model decreases. In the same direction, moving from a 15% probability of failure to a probability of 20% according to graph (9) is again associated with a reduction in problem solving time, but this time the reduction in solving time also reduces the level of optimality of the solution obtained for the objective function. follows Numerical comparison and more detailed investigation shows that in the movement, the probability of stopping from 15% to 20%, the solution time decreases by 0.01%, and the optimality level of the objective function of the problem shows a decrease of 0.3%. Unlike the case of sensitivity analysis, this time the effect of the optimality of the objective function is greater, and according to this argument, the value of 15% of the probability of being stuck in the problem solving process is selected with the help of genetic algorithm.

4. COCLUSION

In this study, a two-level problem was investigated to solve network design problems. For this purpose, genetic algorithm parameters were carefully scrutinized and the following results were obtained to determine the best solution for network design problems.

- The larger the size of the initial population that is used in solving the problem, the time to solve the model for one iteration of the algorithm increases because the solution factors increase and the extent of the algorithm execution increases.

- Increasing the initial population will improve the answer. This is because a more detailed search is performed with a larger number of solving factors in the feasible space.

- Selecting 65% of chromosomes in each generation to perform reproduction operations to move to the next generation has reduced both the optimality of the objective function of the problem and the time of solving the problem.

- The solution time of the model also shows the same 65% as the optimality of the search objective function after the best reproduction probability value.

- The greater the survival probability, the more number of chromosomes of the current generation of the elite have been detected and will be transferred to the next generation in the process of solving the problem in the genetic algorithm.

- The higher the survival probability is, the more chromosomes of the current generation will be transferred to the next generation without any operation, which will naturally decrease the solution time.

- As the survival probability increases, the severity of its reduction effect on the model solution time decreases.

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AUTHORS CONTRIBUTION
This work was carried out in collaboration among all authors.

ONFLICT OF INTEREST
The author (s) declared no potential conflicts of interests with respect to the authorship and/or publication of this paper.

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