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GFRP Elastic Gridshell Structures: A Review of Methods, Research, Applications, Opportunities, and Challenges

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ABSTRACT

Gridshell structures have the potential to develop the construction process of free-form structures, offering numerous benefits. These include the minimum use of materials, light-weighting, the creation of a large span structure, structural efficiency, organic shapes, potential for quick and cost-effective construction, column-free spaces, maximum transparency, sustainability, and ease of deconstruction and recycling. Gridshells, regarding their architectural potential and intrinsic geometric rationality, are well-suited for creating complex shapes. Hence, the properties of gridshells depend on the equivalent pre-stress of the two-dimensional grid that was deformed. The mechanical properties of glass fiber reinforced polymer (GFRP), such as high elastic limit strain, strength, and Young's modulus, can further enhance the potential of gridshell structures. Gridshell structures offer numerous opportunities for constructing double curvature shells. However, they also present challenges, particularly in the design and construction process, while minimizing stress and preventing breakages of elements under the influence of forces. This paper presents a review of GFRP elastic gridshell structures, including design and construction methods. Additionally, a case study of an existing gridshell structure, the Solidays gridshell, is presented. Finally, the opportunities and challenges associated with gridshell structures are discussed.

Keywords: elastic gridshell, form finding, erection process, GFRP, composite material

1. INTRODUCTION

The gridshell structure is a lightweight lattice roof with a large span and double curvature. Gridshells are made of a flat grid without in-plane shear rigidity [1]. Thus, the grid can be deformed perpendicular to its plane and create a double curvature surface. After bracing, their behavior is the same as that of a shell [2]. The strength of these structures is derived from the double curvature form [3]. Comprehensive studies on this subject have been conducted by Bouhaya [4], and Hernandez [5]. Reviews of form finding methods and bracing are presented in [6] and [7], respectively. In the 1970s, the architectural community used this idea for large-scale construction [1] by proposing the concept of an elastic gridshell that is a regular grid of elastic rods [8]. After that, some research focused on case studies [1], [9], [10]. Several investigations proposed computational form finding of these structures [11]–[13]. Gridshell structures offer numerous benefits compared to other structures, such as minimal use of materials, lightweight structure, the creation of large span structures, structural efficiency, organic shapes, potential for quick and cost-effective construction and transportation, low-tech assembly of linear elements, creation of column-free space, maximum transparency, sustainability, and ease of deconstruction and recycling [1], [10], [14]. To maintain the application of these structures, advanced analysis, progress in computer methods for simulating complex gridshells, new techniques for form finding and construction processes, application of machine learning (ML) algorithms in design and optimization processes [15], [16], innovative node clamps, and finding ways to create permanent gridshell structures instead of temporary ones must be taken into account. Many construction projects for curved structures still use conventional methods and practices, such as complex two-dimensional drawings, frameworks, extensive labor requirements, lengthy construction periods, significant material consumption, expensive processes, and consequently, difficult construction processes. A significant advancement in the construction of curved shells is the adoption of gridshell structures. Constructing elastic gridshells is cost-effective, as it reduces the amount of material required for the structure, and it allows for the creation of flexible shapes due to the elastic nature of the materials used. Therefore, the gridshell is one of the structural typologies that enable the creation of various rounded forms at a low cost [17]. The initial gridshells were made of wood due to their low density and an elastic limit strain of about 0.5%, but their weak point was their low strength, reaching a maximum of 30 MPa [18]. In recent years, composite materials such as glass fiber reinforced polymer (GFRP) have been introduced for these types of structures [19]. Since the 1970s, the concept of elastic gridshells has led to notable realizations, such as the Mannheim Multihalle [20] and Downland [21]. The first large-scale timber gridshell, Manheim Multihalle, was constructed in 1976 and has been highly successful, becoming one of the best structures of the 20th century [20]. Two large-scale GFRP gridshells were built in recent years; the Solidays festival in 2011 [9] and Creteil Cathedral in 2013 [19]. Table 1 presents a list of constructed gridshells from the 1960s to the present day.

Table 1. The main constructed elastic gridshells [6]

Name	Year	Location	Span	Number of Layers	Material	Architect	Structural Engineer
Essen	1963	Germany	15m*15m	2	Timber	Frei Otto	Frei Otto
Mannheim Multihalle	1976	Germany	60m*60m	4	Timber (hemlock)	Frei Otto, Carlfried Mutschler & Winfried Langner	Ove Arup & Partners (Edmund Happold & Ian Liddell)
Lausanne Polydome	1991	Switzerland	25m*25m	2	Timber	IBOIS Laboratory	IBOIS Laboratory
Earth Centre	1998	UK	6m*6m	4	Timber (oak)	Grant Associates	Buro Happold
Expo 2000	2000	Germany	72m*35m	2	Cardboard	Shigeru Ban, Frei Otto	Buro Happold
Downland Museum	2002	UK	48m*15m	4	Timber (oak)	Cullinan Architects	Buro Happold
Savill	2004	UK	90m*25m	4	Timber (larch)	Glen Howells Architects	Buro Happold & Engineers HRW
Chiddingstone Orangerie	2007	UK	12m*5m	4	Timber (oak)	Peter Hulbert Architects	Buro Happold
Gridshell Ur Navier	2007	France	24m*6m	2	GFRP	ENPC	ENPC
Solidays festival	2011	France	35m*10m	2	GFRP	ENPC	T/E/S/S
Creteil Temporary Cathedral	2013	France	23m*15m	2	GFRP	Tom Gracy	T/E/S/S

A review of elastic gridshell definitions, GFRP material, bolted joints, and bracing is presented in Section 2. In Section 3 of this paper, the various design techniques and form finding of the gridshells are described, and their convenience is discussed. A review of the construction methods is outlined in Section 4. The analysis of a constructed GFRP gridshell is studied in Section 5. Finally, the opportunities and challenges for GFRP gridshells are discussed in Section 6.

2. GFRP ELASTIC GRIDSHELL

The main advantages of GFRP elastic gridshell structures are their time-efficient and cost-effective construction techniques [5]. The slender curved members create the shape of the gridshell; therefore, the result is an elegant and stylish structure. Additionally, they are sustainable structures due to the minimum use of materials [22]. Figure 1 presents the design criteria for elastic gridshells. This section provides a review of elastic gridshell definitions, GFRP material, bolted joints, and bracing. It also highlights opportunities for the construction of GFRP elastic gridshells in the future.

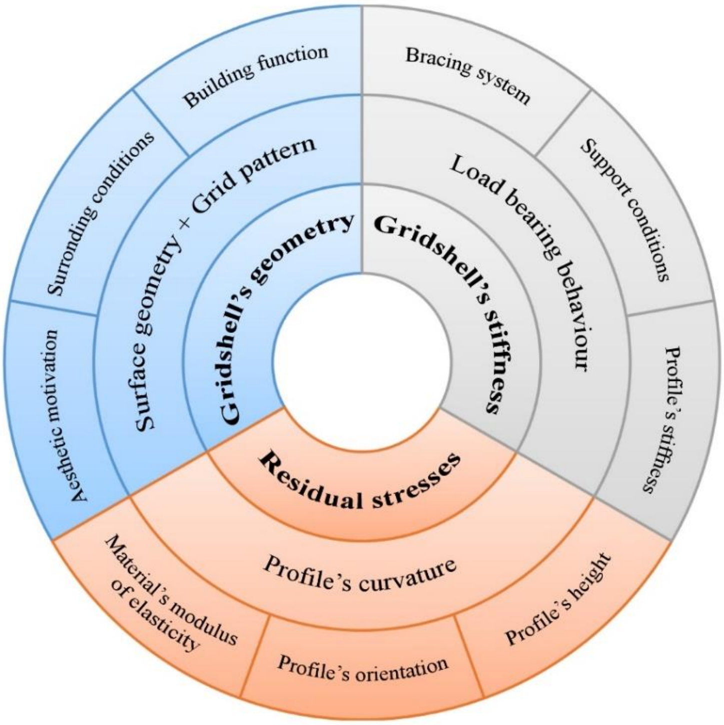


Figure 1. Design criteria for elastic gridshells

2.1. ELASTIC GRIDSHELL

The elastic gridshell is a lightweight lattice roof with a large span that has double curvature and is constructed from a flat grid without shear rigidity [2], [19]. Such a flat grid can be deformed by the loads perpendicular to its plane to obtain the desired shape. the structural behaviors and fabrications of elastic gridshell structures are significantly influenced by the double curvature geometries and the grid patterns [23]. Due to the low density of materials and the large deformations during the construction, the total stress of the slender beams in the GFRP elastic gridshells is mainly induced by bending and can be calculated by Equation (1) as follows:

$$\sigma \approx \frac{M}{I/r} = E \cdot \frac{r}{R} \qquad (1)$$

where σ, M, E, r, I, and R represent the total stress, bending moment, longitudinal Young's modulus, the sectional outer radius, the sectional moment of inertia, and the radius of curvature of the beams, respectively. Thus, concerning the material strength of GFRP, the minimum beam curvature is limited to be no smaller than the allowable values, as shown in Equation (2). according to this equation, the shape of the elastic gridshell can be optimized in the schematic design stage.

$$R_{\min} > \frac{E \cdot r}{\sigma_{\max}} \tag{2}$$

where $R_{\min}$ and $\sigma_{\max}$ represent the minimum radius of beams curvature and the maximum stress, respectively.

2.2. Glass fiber reinforcement polymer material

Several studies have been conducted about the best material for elastic gridshells. Douthe et al. [18] studied new materials that are better than wood for constructing the elastic gridshells. Kotelnikova-Weiler [24] investigated the long term behavior of pultruded GFRP bars under torsion and bending stresses. Compared with wood, GFRP profiles demonstrated a higher elastic limit strain of 1.5% and higher strength limit over 400 GPa. Thus, a larger curvature of freedom can be obtained in gridshell structures made of GFRP. Moreover, there is a linear dependency between the buckling load and Young's modulus, so a higher buckling load of GFRP gridshells might be obtained due to the higher Young's modulus, which is 25-30 GPa [25]. Furthermore, they exhibit excellent resistance in aggressive atmospheres such as UV, seawater, and acid air. Additionally, they require less maintenance expenditure during the service period.

2.3. Bolted joints

There are three types of connectors in GFRP elastic gridshells, including the end-to-end connector, the swivel connector, and the ground anchorage. These connectors are shown in Figure 2, respectively. The end-to-end connectors join GFRP profiles to achieve a longer beam. The critical point is to keep the mechanical properties of the assembled GFRP beams for both tension and bending the same as the primary GFRP profiles after connecting [26]. The swivel connectors attach two beams in different layers, allowing in-plane rotations and preventing translation and out-of-plane rotations. Therefore, the orientation of the connector is unconstrained, and as a result, only nodal forces are transferred between beams [27]. The ground anchorages fix the member end to the ground after the erection process [26]. The end part of each beam on the edge of the structure is fixed to its correspondent anchorage, and simultaneously, the desired shape of the grid will be obtained [28].

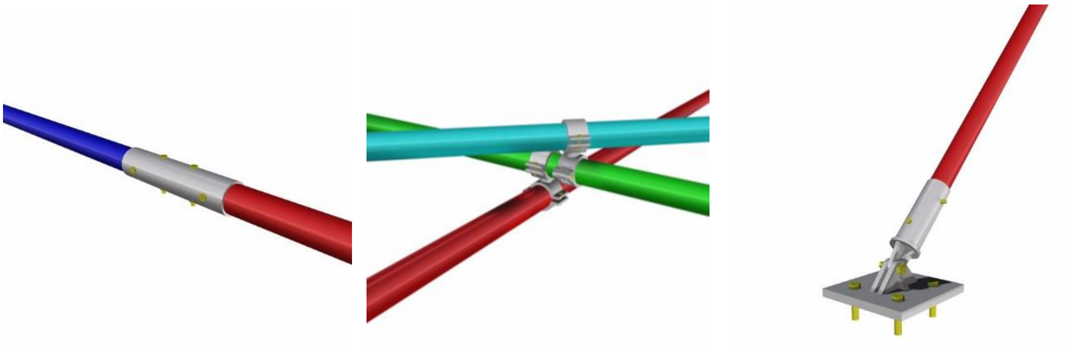

(a) **(b)** **(c)**

Figure 2. (a) Assembly used to join two beams **(b)** swivel connector **(c)** Pin anchorage for beams

2.4. Bracing

Bracing is an essential step because without bracing, the gridshell has no in-plane shear resistance. For GFRP elastic gridshells, a diagonal layer of beams attached to the two-layer grid is commonly used for bracing. The bracing changes the shape of the grid pattern from a deformable quadrangle to two rigid triangles, and after that, the structure gains in-plane shear rigidity and behaves like a shell. Related research indicates that after bracing, the stiffness of the structure might be about twenty times higher than before bracing, and the structure obtains its full mechanical properties [9]. Hence, the third layer of beams must be set up on the deployable two-layer gridshell; thus, this step is the meticulous one. Although bracing would not change the morphology of the structure significantly, it may slightly modify it due to the self-weight and the bending of the diagonal beams.

2.5. Discussion

The described GFRP gridshell structures highlight several essential factors for constructing curved shells. The construction of this type of structure reduces the time and labor required on-site. It is worth noting that the costliest stage is bracing, as it accounts for 30% of the labor work time. The use of a small amount of material and the ease of deconstruction and recycling are other key themes emphasized by these structures. Additionally, it has been shown that this technique can create lightweight and sustainable structures due to the minimal use of materials. Moreover, the high residual stress resulting from the bending process is a significant structural challenge that can be mitigated by using GFRP material because of its high strain limit.

3. DESIGN OF GFRP ELASTIC GRIDSHELLS

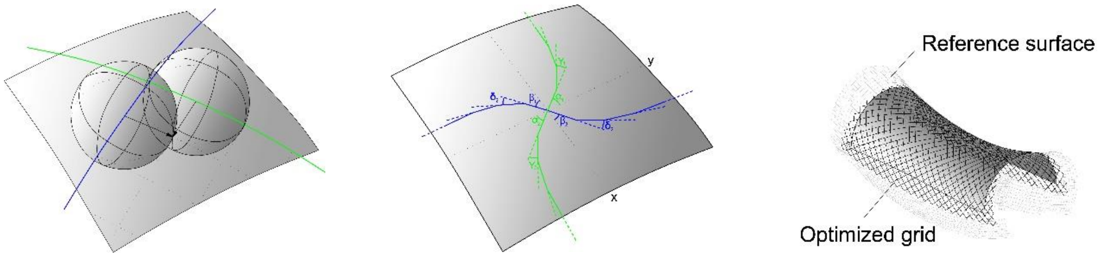
The design of GFRP elastic gridshells mainly consists of two phases; the first one is the geometry design, and the second one is the structural analysis. In the geometry design phase, the grid pattern of the as-built gridshell and the corresponding two-layer flat grid needs to be determined. In the last century, physical models were commonly employed for the geometry design of the elastic gridshells [29], [30]. In 1676, Robert Hooke concluded that by reversing the shape of a hanging chain, the stable configuration of the arch could be achieved under the same loading [6]. Otto and Rasch [31] used this technique for the form finding procedure. In this method, an optimal structural design will be obtained under the influence of gravity while considering a set of boundary conditions and specific material [32]. Some of the most recent studies have highlighted the form finding process. Su et al. [33] offered a novel shape generation method for complex gridshells by integrating the multi-objective optimization and numerical inverse hanging method. Xiang et al. [34] proposed an innovative approach for the design and analysis of GFRP elastic gridshells, combining the form finding analysis and the lifting construction simulation. Furthermore, in their subsequent study [35], a form finding method was developed to predict the nodal forces, deformation, and bending moment of biaxial symmetrical gridshells.

3.1. Geometry design

In the design of GFRP elastic gridshells, the existing design approaches can be classified into two categories, depending on whether they consider the static equilibrium of the gridshell members during the determination of the grid pattern. The first category, in which static equilibrium is not involved, derives the grid patterns of the gridshell based on a designed reference surface, using geometric and mathematic principles. Consequently, the corresponding flat grid can be determined according to the grid pattern. In this approach, the surface of the as-built gridshell needs to be determined through further numerical analysis, which simulates the shaping of the determinate flat grid. In the second category, the grid pattern of the gridshell is achieved through a process called form finding, which takes into account the mechanical properties of the members, shaping loads, and boundary conditions. These approaches primarily rely on structural analysis software based on the dynamic relaxation method or finite element method (FEM) [36], [37].

3.1.1. Design approaches based on geometric and mathematic principles

Otto [1] utilized a uniform mesh grid with square cells and presented an index of the obstacles, such as singularities and overlapping. Afterwards, a method called the compass method is presented to apply the Chebyshev meshes on curved surfaces. Bouhaya et al. [25] established an alternative to the compass method by numerically dropping a grid onto the curved shape. The advantage of the proposed method, compared to the compass method, is that it considers the real mechanics of the grid. Hernandez [5] presented three methods for designing elastic gridshells: the compass method, genetic algorithms, and variational principles, as shown in Figure 3.



(a) (b) (c)

Figure 3. Design approaches: **(a)** Compass method; **(b)** Genetic algorithm; **(c)** Variational principles [5]

The first method presented by Hernandez [5] is the compass method. Bouhaya et al. [38] carried out the compass method by generating a large number of discrete guidelines on the surface while the process was controlled by a vector of angles. Afterwards, the method was coupled with the genetic algorithm to find the meshes with the minimum curvature of the beam elements. Pone et al. [39] recommended a similar tool that was developed by Bouhaya et al. [38] and Du Peloux et al. [40].

In the first step of the compass method, two secant curve are created on the curved surface. These curves define the boundary of four quadrants. Then, each part is subdivided by a compass of constant distance. In the end, the quadrants of two sequential half parts are meshed, considering the same compass distance. The process is shown in Figure 4, and Figure 5 illustrates the flowchart of this method.

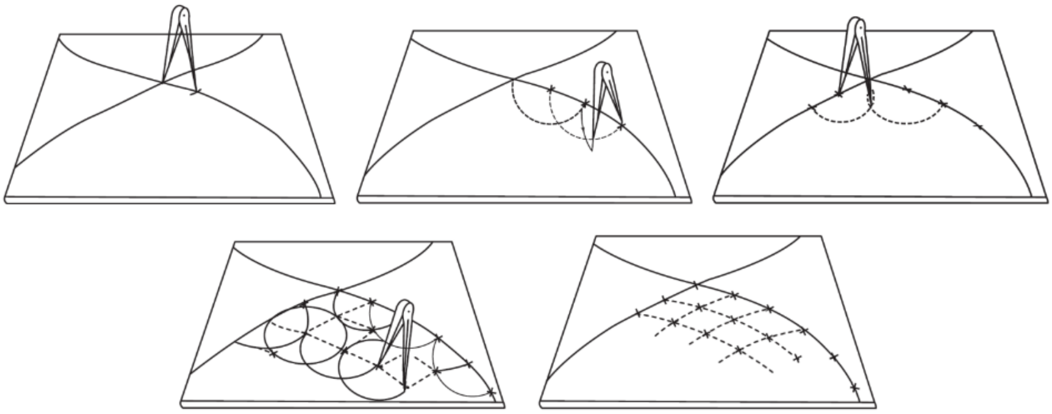


Figure 4. *Construction of the grid using the compass method [1]*

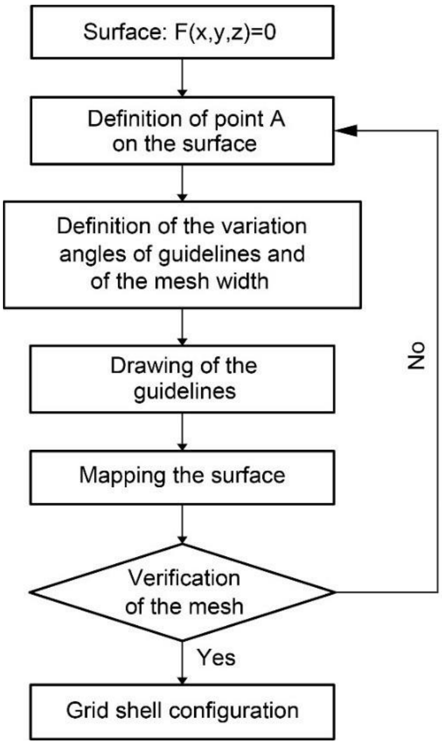


Figure 5. Flowchart of compass method [38]

Bouhaya [4] used an optimized profile curvature to determine the grid on the final surface. This method employs the compass method to map several potential options of grids on the final surface. Then, they are compared based on the curvature of the profile and the best one is chosen using stochastic genetic algorithms.

Hernandez et al. [41], [42] defined variational principles to determine the grids with optimal curvature on the reference surface. This was achieved by introducing penalty energy. As a result, the mesh is capable of deviating from the defined shape. This method aims to establish external functions to minimize the value of different factors related to grid properties, such as the distance to a reference surface, the curvature of the profiles, and the deviation of edge lengths, while considering the desired mesh length. The benefit of this method is that it can establish a different grid configuration based on the structural requirements by applying a weighting factor.

3.1.2. Design approaches based on form finding

The formation of the elastic gridshells began in the 1960s, and significant progress has been observed in the computational form finding of the gridshells during the last few years [10]. Wehdorn-Roithmayr [43] presented approaches to the geometric design of the gridshell as follows: the first one is called Funicular methodology, which uses the inverting shape of a hanging chain model. The second one is the analytical method that defines a surface and then applies a grid of nodes and lines on that surface. The third option is an integration of the two previous methods, which was used in the Downland gridshell [44], [45].

Adriaenssens et al. [46] proposed a 3-DOF discrete beam element for form finding of elastic rods with an isotropic cross-section. Adriaenssens and Barnes [47] noted that this element has greater stability relative to the previous 6-DOF element. D’Amico et al. [36] and Poulsen [48] applied the 6-DOF beam element for the form finding of the gridshells. Du Peloux et al. [49] and Lefevre et al. [50] offered a 4-DOF element regarding the torsion and bending behaviors of slender rods.

Choosing an appropriate method for form finding is influenced by various factors such as the properties of the material, the type of structure, limitation stress and displacement to absolute values, avoiding buckling and excessive creep, construction requirements and costs, reasonable lifetime, aesthetically pleasing design, boundary conditions and load cases [51]–[53]. Hernandez [5] proposed several form finding approaches, including form finding with a predefined flat grid, form finding with vertical shaping springs, form finding with vertical shaping forces, and the least strain energy method, as demonstrated in Figure 6.

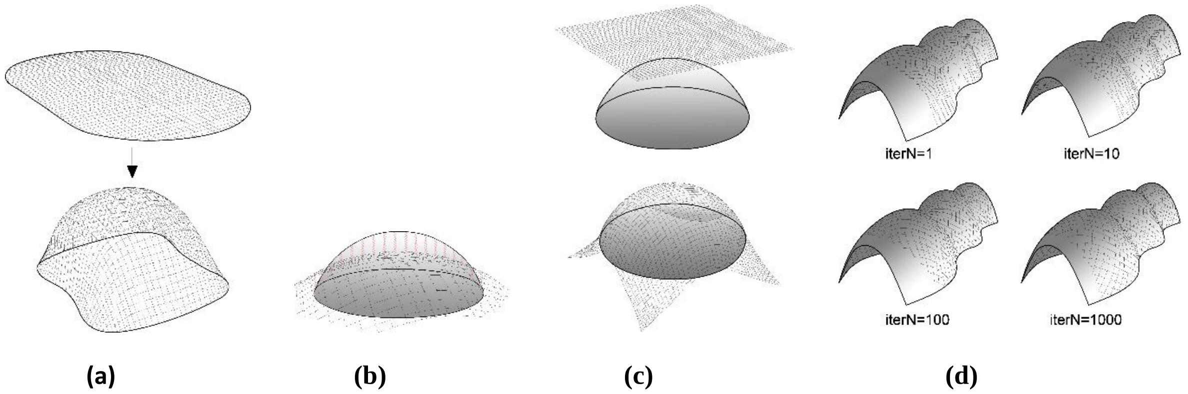


Figure 6. Design approaches: **(a)** Form finding with a predefined flat grid; **(b)** Form finding with vertical shaping springs; **(c)** Form finding with vertical shaping forces; **(d)** Least strain energy method [5]

In the case of form finding with a pre-defined flat grid, the first flat pattern of the grid defines the final geometry of the structure. Day [54] proposed the dynamic relaxation method, which can be used to calculate the static equilibrium of forces. Barnes [55] and Barnes and Wakefield [56] have made further developments. In their approaches, a pre-defined elliptic flat pattern of the grid is used. initially, the shaping of the grid was induced by external springs; however, later upward loads at the grid nodes were applied as shaping forces. Another method is form finding with out-of-plane shaping forces. In this case, Kuijvenhoven [57] and Kuijvenhoven and Hoogenboom [58] proposed a design approach that includes an iterative process to create a grid pattern. In this method, a flat grid with the free-boundary condition is pushed towards a reference surface by a system of vertical shaping springs. A second static equilibrium is subsequently recognized without shaping forces by the algorithm based on dynamic relaxation method [5]. Bouhaya et al. [25] presented the same design approaches, while allowing for friction between the reference surface and the grid. In the case of the least strain energy method, Li and Knippers [59] presented a design method in which the flat grid is placed over the reference surface by applying the constraint force at the nodes. Vectors pointing from the grid nodes to the closest point on the reference surface define the constraint forces. These forces are proportional to the distance between the grid node and the respective point on the reference surface.

3.2. Structural analysis

Following the form finding, the structural behaviors of the obtained gridshell must be checked by considering the external loads. As specialized digital design tools for elastic gridshells are currently unavailable, general-purpose commercial analysis software is commonly chosen by designers and researchers for structural analysis [40], [60]. These software packages are primarily based on the theories of FEM and the dynamic relaxation method [18], [61]. This section presents a review of the applications of these two analysis methods.

3.2.1. Finite element analysis

Finite element analysis (FEA) is the most popular method for structural analysis in both design and research. In the early studies on gridshells using the FEA method, the manufacturing process was not considered, and the compression and rigid members of gridshells were taken into consideration [11]. Mesnil [62] conducted a study on the gridshell to find a way to reduce the displacement. As a consequence, the expected shape of the buckled gridshell members was described, and theoretical solutions for the failure of large-scale elastic structures were provided, which included the shape of a buckled beam [63]. Mesnil et al. [64] investigated the failure mode of GFRP elastic gridshells with various grid densities, as well as the influence of pre-stress on the buckling capacity, by using the FEA. Lefevre et al. [13] studied the buckling of GFRP elastic gridshells, considering the effects of the eccentricities at the connections induced by the connectors on the platform of FEA. The advantages of this method are rapid improvement and development over the last few years, efficiency, the ability to solve a wide variety of problems, and dealing with different geometries [6]. Abaqus [25], [65], Sofistik [11] and ADINA [62], [66] are non-specialized software packages that are suitable for gridshells.

3.2.2. Dynamic relaxation method

The dynamic relaxation method is considered an efficient numerical technique for analyzing the equilibrium conditions of highly nonlinear systems. Due to this characteristic, it is widely used in the form finding of flexible structures, such as the membrane structures and cable-net structures, since its introduction into structural analysis in the 1970s. Furthermore, it has been improved to be applicable in the analysis of these structures under external loads. Several studies have been performed on the dynamic relaxation method [47], [67], and some investigations have proposed gridshells [68], [69]. Barnes [55] developed this method for pre-stressed structure.

Researchers at the Navier Laboratory in France compared the structural behaviors and the required time in the analysis of a GFRP elastic gridshell based on the dynamic relaxation method and the FEM. The results show that the mechanical indexes are comparable, and the dynamic relaxation method presented a more efficient calculation [70]. Subsequently, they applied the dynamic relaxation method to the structural analysis of the Solidays gridshell and the Créteil Temporary Cathedral, which were respectively built in 2011 and 2013, under permanent bending and external loads such as snow load and wind load [28], [49].

3.3. Design codes and standards

In the field of building construction with composite materials, three popular standards and literature are widely used as references: 1) *Eurocomp Design Code and Handbook* [71], 2) *Guide for the Design and Construction of Structures made of FRP Pultruded Elements* [72], and 3) *Composites for Construction: Structural Design with FRP Materials* [73]. The primary literature is a pre-standard intended for the structural design of buildings and civil engineering structures using GFRP composites It is consistent with the design approach of the Eurocode and is considered a reference design code for GFRP materials. The second one is a non-binding regulation proposed by the National Research Council of Italy. It specifically deals with structures made of GFRP pultruded elements. The third literature is a reference book that provides a comprehensive review of the application of FRP in civil engineering. It also includes design procedures in accordance with published structural engineering design codes, guides, and specifications related to the application of FRP in engineering structures. For structures made of GFRP members, such as the GFRP elastic gridshell, design considerations can be derived from these three sources of literature.

3.4. Discussion

The structural design of gridshells requires both form finding and analysis based on the design codes. An appropriate form finding method must be chosen based on the type of gridshell. In terms of design based on mathematical principles, the variational principles method does not utilize the constraint surface geometry. Additionally, the optimization of the curvature of the profiles has not been taken into account by the compass method [12]. In the case of the design based on the form finding, all methods except the predefined flat grid method utilize a reference surface geometry. Upward loads or external springs act as shaping forces for the first method, while the other method incorporates shaping forces such as vertical forces, constraint forces, or vertical springs defined straight or vertically between the reference surface and the grid [5]. Currently, numerical methods begin with an inefficient form and then minimize different forces to achieve structural efficiency [6]. FEA is commonly used to determine a suitable shape for a gridshell that can withstand external loads. However, this method is computationally expensive. Therefore, a computationally efficient numerical tool specifically designed for gridshells, with the ability to explore more complex structures and accurately represent the mechanical behavior of gridshells, is highly necessary. Finally, the structure should be analyzed based on the mentioned building code to verify safety. For GFRP elastic gridshells, which are flexible structures involving large deformations, the design methods presented in the mentioned references of design codes and standards may not be sufficient to ensure strong safety. Therefore, a design code specifically tailored for GFRP elastic gridshells, similar to the codes developed for steel spatial structures, is critical in promoting such structures, and further studies are required for the future.

4. ERECTION PROCESS

Due to the high bending stress that may be produced by tight curvatures and the point loads on the grid during construction, the erection process for elastic gridshells is critical and plays an essential role [52]. Generally, most wooden gridshell structures might encounter fractures in several of their elements during their erection process. Due to the superior material properties of GFRP, member fractures will be less likely to occur. Nevertheless, reducing or avoiding these failures is still an important point to consider. Thus, choosing a proper erection method is essential, and the size and shape of the gridshell and the site conditions should be considered. Alessandro and Alberto [74] conducted a review of the construction methods of gridshell structures with an emphasis on their features and deficiencies. Quinn and Gengnagel [10] offered five methods for the erection process of elastic gridshell structures, which are “pull up”, “push up”, “ease down”, “inflate” and “by constraint”. The five erection methods, as shown in Figure 7, are introduced respectively in this section.

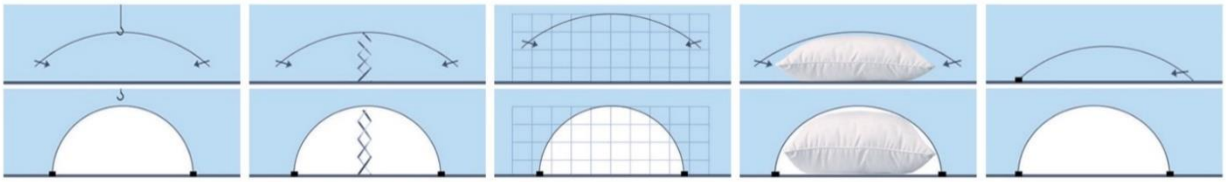

(a) (b) (c) (d) (e)

Figure 7. Erection methods for elastic gridshells: **(a)** Pull up; **(b)** Push up; **(c)** Ease down; **(d)** Inflate; **(e)** By Constraining

4.1. Pull up

The “pull up” method is a quick way to construct GFRP elastic gridshells, but it creates large loads at the lifting points where the cables are attached to the grid, which may cause fractures in GFRP members. Therefore, many lifting points are needed to decrease the risk of overstressing the material during the procedure [48]. While the vertical forces are inserted from outside the plane by the cables, the horizontal components of the cable forces may cause an in-plane pressure inside the structure or additional stresses on the members, increasing the risk of buckling during the erection. By implementing the “pull up” method, no workers need to be under the structure during the erection process, making it a safe method [48]. However, since the pull up process is significantly influenced by wind, calm weather is required for construction. Moreover, this method is not suitable for large gridshells due to the limitation of the working radius of the crane [10].

4.2. Push up

This method is suitable for small structures and does not require new technological devices. It has been used for a few small pavilions so far. However, there is still no certainty about the safety of this method due to its limited application [48]. The method involves using a jacking tower that can be vertically lifted using forklift trucks. These trucks can accommodate the necessary lateral translations of the anchoring points involved in the lifting. A crucial aspect of this method is to identify specific points for anchoring to prevent collapse [10].

4.3. Ease down

The “ease down” method begins by creating a flat grid on a raised level provided by the use of a mechanical framework or modular scaffolds [10]. In this method, the grid is gradually displaced downwards and allowed for lateral movements. This method is suitable for avoiding the lifting process. For this purpose, the flat gridshell should be constructed at the final level of the structure. During this process, careful monitoring of the structure is necessary to prevent potential hazards resulting from unexpected stress conditions. For instance, significant local stresses may arise in the attachments of temporary support, potentially leading to a collapse of the structure [48].

4.4. Inflate

This method has not yet been used for gridshells, but it can be extended to gridshells. The steep surfaces of a pneumatic cushion with small horizontal contact surfaces tolerate less vertical external force with low static pressures compared to the flatter zones of a pneumatic cushion [52]. This point is suitable for the erection process, but it is inappropriate for the final geometry. In this method, The height of the gridshell should be set at a higher height than the final shape, and then the ends of the beams can be lowered to fix them in their supports by deflection, regardless of the cushion type [10]. Onate and Kroplin [75] introduced this method entirely in their book, while the focus of the book is on concrete skins, there are many findings related to the gridshells. Quinn [76] proved that this method has several advantages, such as structural robustness, speed of erection, and architectural qualities.

4.5. By constraint

This method is suitable for elastic gridshells that have two symmetrical boundary lines. Modifications and forming the final shape are made easy by this method for such gridshells. The advantage of this method is that the forces need to be applied only on two sides. However, special care is required for the fixed points and the points where forces are applied to avoid concentrated stresses on individual elements and to distribute the forces uniformly in the structure, thereby producing a harmonious stress state. No structure has been built using this method so far, and it is presented as a theoretical method that requires experiments and research. This method is practical due to its speediness and the low-tech requirements in facilities [52].

4.6. Discussion

The five erection methods presented above can be adopted for the construction of GFRP elastic gridshells on many occasions, and if necessary, these methods can be used in combination. The “pull up” and “push up” methods, which provide fast and economical construction process, are applicable for small structures. The “ease down” and “inflate” methods can be implemented for large-scale gridshells; they reduce the risk of member fractures since the grid is supported over a large region rather than at several points. The “by constraint” method can be applied to structures that have symmetrical boundaries where large-scale construction devices are not available.

5. CASE ANALYSIS

In 2011, a gridshell was built in Paris to provide shelter for up to 500 people at the Solidays Festival, as shown in Figure 8. The half-peanut-shaped gridshell covers an area of 280 m^2 with a length of 26 m, a width of 15 m, and a height of 7 m. The pultruded GFRP circular tubes in the structure have a longitudinal Young's modulus of 25 GPa and a limit stress of 400 MPa. A PVC membrane of approximately 500 m^2 is used for cladding. The material usage of this structure is about 5 kg/m^2, making it a lightweight structure. This structure was deconstructed after the three-day festival. The structure was constructed by only ten people who worked on site in a few hours, showcasing the advantages of fast and easy construction for such structures.

Figure 8. GFRP elastic gridshell for Solidays Festival [17]

Du Peloux et al. [40] proposed the design process of this structure. The compass method was used for the form finding process [38]. Tayeb et al. [28] proposed an analysis in different cases for this structure. They calculated the maximal displacement, $U_{\max}$, by considering the index *i*, which characterize the node *i*, using the following formula:

$$U_{\max} = \text{Max}\left[\left(x_i^2 + y_i^2 + z_i^2\right)^{\frac{1}{2}}\right] \tag{3}$$

In addition, the mean displacement $\overline{U}$ was calculated using the formula below, where N_t represents the total number of nodes:

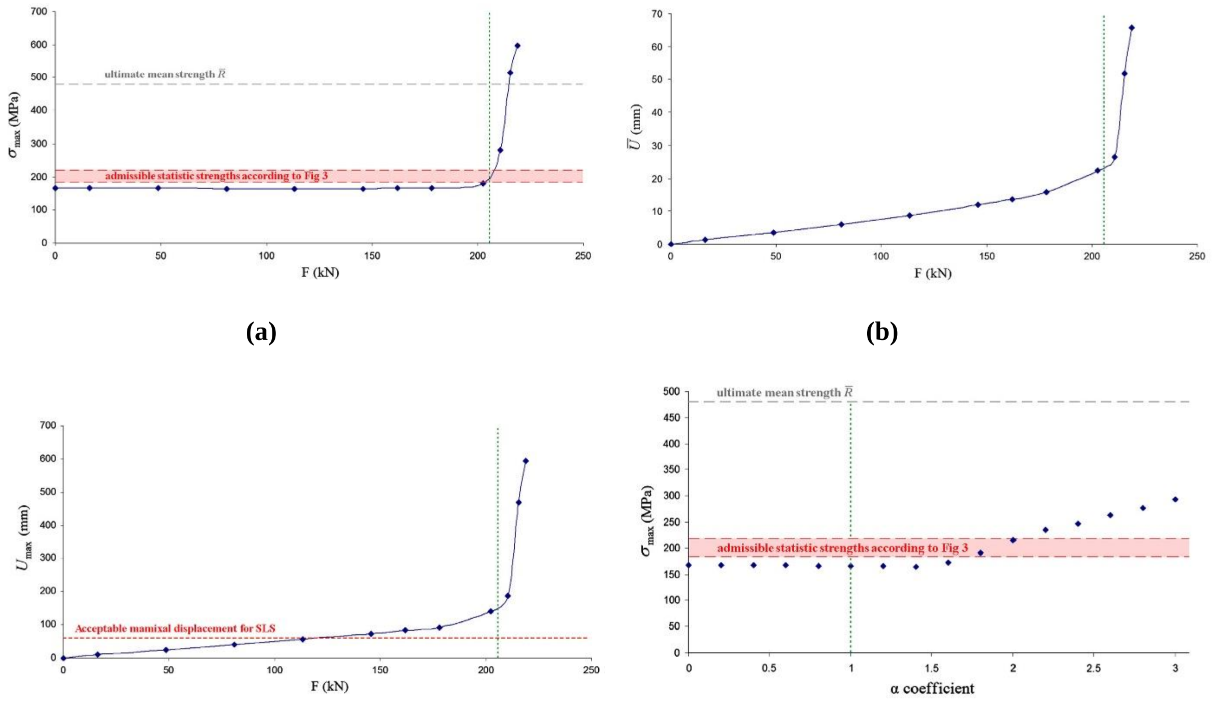
$$\overline{U} = \frac{\sum_{i=1}^{N_t} \sqrt{x_i^2 + y_i^2 + z_i^2}}{N_t} \tag{4}$$

In their study, the ratio of stress over strength β_i is the essential output that can be calculated for element *j*, with considering $\sigma_{\max}^j$ as the maximal stress and R_d^j as the strength of the element *j*, as follows:

$$\beta_j = \frac{\sigma_{\max}^j}{R_d^j} \tag{5}$$

The results are shown in Figure 9. As can be seen, it exhibits elastic behavior initially and then undergoes buckling, reaching the weakest strength of a critical beam very quickly. The green dotted line indicates the highest point of statistical strength. Figure 9d presents the results for the critical ultimate limit state (ULS) case, illustrating that buckling occurs when $\alpha \approx 1.5$. For more details, please refer to [28], [77]. It was concluded that this gridshell displays pseudo-ductile behavior. Additionally, the load cases of

Eurocode, including the ultimate limit state (ULS) and the serviceability limit state (SLS), do not result in buckling even after breaking several elements [28].

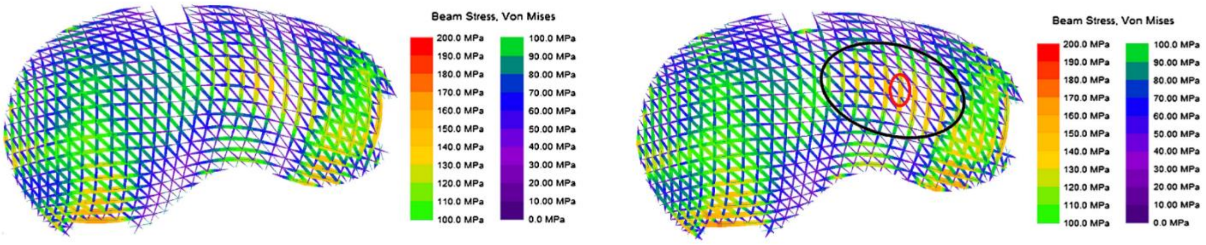


(a) **(b)**

(c) **(d)**

Figure 9. a) Maximal stress (*MPa*) versus vertical load (*kN*), for an elastic gridshell. **b)** Mean of nodes' displacement (*mm)* versus vertical load (*kN*), for an elastic gridshell. **c)** Maximum displacement (*mm*) versus vertical load (*kN*), for an elastic gridshell. **d)** ULS load margin before buckling. [28]

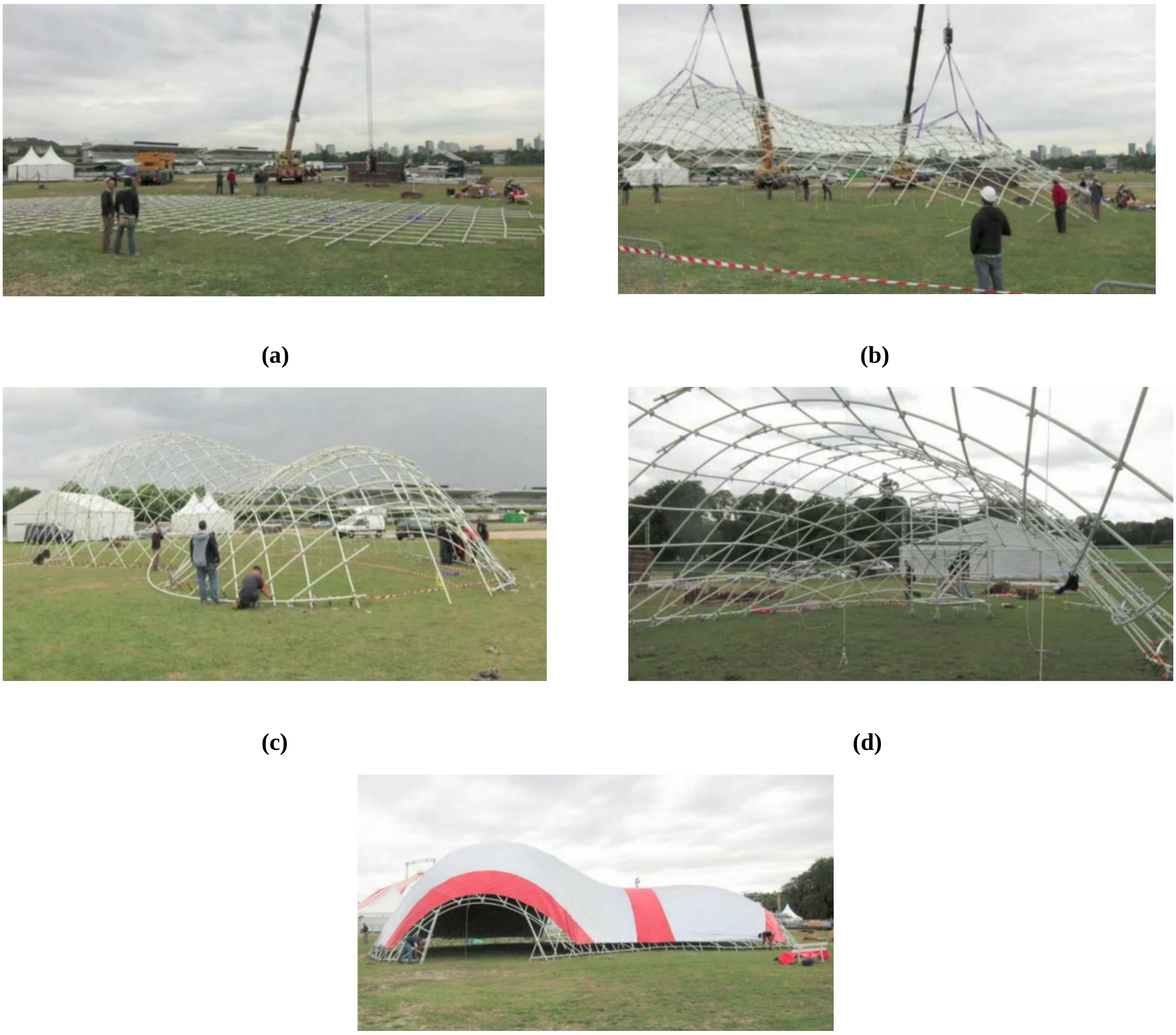
For simplicity, the load chosen in their study is the snow load. As shown in Figure 10a, the members of this gridshell are pre-stressed. Then the ULS force is applied, and the tension slowly changes in the region shown in black in Figure 10b; tension slightly increases [28]. Subsequently, the elements located within the red circle in Figure 10b were broken. After the breakage, the stress transferred to neighboring elements due to the redundancy of the gridshells [28], [78].



(a) **(b)**

Figure 10. a. Stress chart on beams resulting from forming only;
b. Stress chart on beams resulting from both forming and ULS load [28]

The pull up erection process was implemented to construct this structure. Figure 11 illustrates the erection process of the Solidays gridshell.



(a) (b)

(c) (d)

(e)

Figure 11. Construction process for the Solidays pavilion [79]

6. OPPORTUNITIES AND CHALLENGESO

GFRP elastic gridshells offer several opportunities and challenges associated with the different issues such as geometry, material, time, weight, environment, costs, manufacture, temperature, design, and optimization. These opportunities and challenges are discussed in this section.

6.1. Opportunities

GFRP elastic gridshells are complicated structures that present several opportunities, and researchers are actively advancing their investigations in this field. These opportunities include geometric flexibility, short construction time, lightweight structures, environmental advantages, and human factors.

6.1.1. Geometric flexibility

Currently, designers tend to design more creative forms, such as double-curvature shells. Thanks to the high deformability and strength of GFRP materials, as well as the creative swivel connectors, the flat grid can deform significantly to achieve various free and special three-dimensional surfaces. Hence, the GFRP elastic gridshell structure provides the opportunity to create a double-curvature surface through a fast and cost-effective process, which is advantageous compared to traditional techniques.

At the same time, the GFRP elastic gridshell offers the advantage of structural efficiency, i.e., the use of fewer materials for a large span, compared to other types of reticulated structures. Furthermore, the low density of GFRP materials makes the elastic gridshell an extremely lightweight structure in comparison to conventional shell structures, such as concrete shells or reticulated steel shells. Figure 12 illustrates this concept, where a portion of a shell with a high amount of material can be replaced with a module of a gridshell, resulting in material reduction.

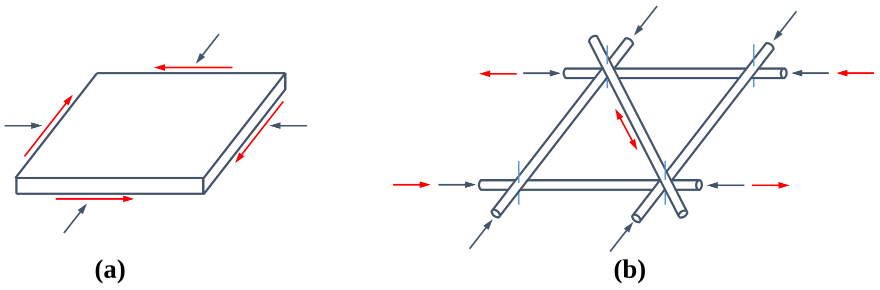


Figure 12. Structural action on (a) a continues shell (b) a gridshell element.

These structures demonstrate high-performance on a large scale when utilizing composite materials [19]. GFRP, as a composite material, is employed in the construction of gridshells due to its flexibility [26], [80]. In addition to flexibility, these materials possess high deformability and strength [9]. Furthermore, the existing gaps between the GFRP tubes provide numerous possibilities for building penetration or lighting [81].

6.1.2. Fast construction

In the construction industry, the longer duration of the construction process will result in higher costs due to labor and equipment expenses. The fast construction of GFRP elastic gridshells effectively reduces expenditures during the construction stage. As mentioned previously, the final form of the gridshell is constructed from an initial flat grid, which offers benefits such as standard profiles, standard connection nodes, and planar geometry [49]. Consequently, the difficulty of construction decreases, allowing GFRP elastic gridshells to reduce the construction time [5]. Additionally, their on-site assembling is a quick process [82]. Therefore, gridshells are applicable for rescue operations during emergencies, such as natural disasters [81]. The demand for large shelters in refugee or disaster-stricken areas, for purposes such as medical treatment, social convalescence, and religious gatherings, is high worldwide.

The construction process of the gridshell, which includes marking nodes, cutting pipes, preassembling, sleeves, swivel couplers, and anchorage, can be easily automated. GFRP tubes can be sliced, milled, and drilled quickly. Small robot arms are capable of performing these tasks faster and with greater accuracy [61]. Typically, 23-35% of the man-hours are spent on connection installation. Therefore, the connection can be redesigned to enable faster fastening and positioning. Additionally, the use of transportable modules can minimize on-site work and subsequently reduce the required time. Bracing is also a time-consuming process, accounting for approximately 30% of the man-hours. One solution to overcome this drawback is to utilize a bidirectional cable network for bracing while the grid is still flat and at ground level. Another possibility is to empty a thin fiber-reinforced concrete skin for bracing. In this case, the structure can serve as a permanent building [7].

6.1.3. Lightweight structures

Lightweight structures are contemporary and necessary for the future from the social, ecological, and cultural points of view. Lightweight structures are popular among researchers [70]. The gridshell is a free-form and double-curved shell made up of a net-like grid, aiming to bear the loads by utilizing the resistance of form and grid configurations. Such a gridshell is known as a form-active lightweight structure [83]. Due to the use of low-weight materials and high strength, it is a lightweight structure that can be easily bent and twisted. Lightweight structures utilize optimum materials strength, making them efficient in terms of materials. The self-weight of the elastic gridshell is about 5 kg/m^2 to 20 kg/m^2, ensuring the lightweight nature of the structure [62].

There are three main reasons for using lightweight structures. The first one is from an ecological aspect, which involves reducing the amount of material and consequently decreasing the environmental impact of a building. Additionally, they can be easily deconstructed and recycled. The second one is from an economic point of view, which adds value to society due to the use of qualified engineers. The last reason is from a cultural aspect, which involves utilizing transparency and lightness that is better than heaviness. Therefore, lightweight structures, especially elastic gridshells, are essential in the construction industry.

6.1.4. Environmental advantages

Frei Otto [1] defined a gridshell as a structure that uses a low amount of material to decrease environmental impacts while considering social and environmental sustainability. As GFRP elastic gridshell structures can provide a large non-column space with a fast and cost-effective construction process, they have the potential for use in crisis conditions. For example, there is a demand for large shelters for medical treatment, social convalescence, and religious gatherings in refugee or disaster-stricken areas around the world [76].

Nowadays, the demand for sustainable structures has increased. Selecting the materials and systems according to the life cycle assessment and environmental impact is one strategy for constructing an environmentally friendly structure. GFRP elastic gridshell is an excellent response to this issue. From the material aspect, the manufacturing of GFRP materials is more eco-friendly than the production of conventional construction materials like steel and concrete. Additionally, GFRP materials have excellent resistance to environmental attacks such as corrosion and UV, which can reduce maintenance requirements during the life-cycle of the structure [80].

Moreover, one environmental benefit is the ease of deconstruction. Consequently, under good maintenance, the components of the GFRP elastic gridshell can be reused in other projects. Furthermore, thermal insulation and penetration of light, benefited from appropriate claddings, consist of the concept of “low carbon” in building design nowadays. Based on the mentioned points, this kind of structure has several ecological advantages in comparison with other structures.

6.1.5. Human factors

It is clear that a structure built for people should be safe, even in unexpected situation [28]. The gridshell structure is a type of structure with low safety requirements [84]. This structure can serve as a safe shelter for people due to its pseudo-ductile behavior, which causes large displacements that warn people before collapse. It has been shown that a high level of redundancy improves resistance to collapse [28]. Since workers do not need to be under the structure during the erection process, the construction method is safe [48]. Moreover, among the various methods of erection, the pneumatic formwork requires less workforce and provides a safer working environment [85]. As stated about Solidays structure, voluntary workers collaborated during the construction process [19]. Local people can carry out the fast, cost-effective, and low-skill construction process, making the GFRP elastic gridshell suitable for the humanitarian relief operations. It is important to note that the design of this structure should be done accurately to minimize failures and errors on-site and remove dangers for humans. Since the gridshell structure does not require any framework and the main part of the construction process is done by cranes, it requires less manual work compared to previous methods.

6.2. Challenges

Currently, GFRP elastic gridshells, despite being lightweight and having the ability to cover large spans, have rarely been used by society because of existing challenges such as costs, manufacturing process variability and limitations, design methodology and digital workflow, design and optimization of structures, temperature, and moisture. It is evident that after solving these problems, they have the potential to be used more widely as both temporary and permanent structures in the building industry.

6.2.1. Costs

The cost of a GFRP elastic gridshell structure will be influenced by expenses for the design phase, material, joint elements, erection process, bracing, human labor, and other necessary equipment. The economic advantages of the gridshells are due to the low material quantities required to cover a large space, a quick construction process, cost-effective transportation, and low-tech assembly for the elements [10]. While gridshell structures offer cost-effective processes, there is still potential to reduce the costs.

In the case of roof structures, the gridshell is one of the most cost-effective structures due to the small number of elements and connections [80], [86]. As a result, fewer human laborers are needed. The costs of human resources are estimated to account for 15-50% of the total cost of constructing a building. Therefore, reducing the number of workers will significantly decrease the construction cost.

Additionally, the cost of GFRP is reasonable compared to other composite materials [62]. However, there is still potential to further reduce the cost of the materials by making changes in the production method or eliminating unnecessary mechanical characterization of the material. Moreover, standard swivel scaffolding elements available in the market can be used with standard tube diameters, which provides cost benefits.

The erection process is the most critical step because it might cause breakages in the elements. Repairing the broken elements can add additional costs to the structure. Therefore, choosing an appropriate erection method that minimizes the number of ruptures is essential for reducing expenses. The construction of a gridshell is a fast process, which decreases the number of construction days on-site and reduces the need for human labor and equipment for an extended period. As a result, the overall cost is reduced. Furthermore, the bracing process should be completed at the height, but it currently wastes approximately 30% of the work hours. The construction process can be made shorter by simplifying the nodal connectors and bracing process.

6.2.2. Manufacturing process variability and limitations

Gridshell structures present some advantages in the case of manufacturing. A double curved form is constructed of a flat grid, which is made of slender elements. Since the initial surface is flat instead of three-dimensional, its connection and manufacturing can be completed more easily [49]. However, finding the final position of the joints is still an unsolved problem [74]. Obviously, the ease of the manufacturing process, which leads to the desired form, is one concern of the designers.

A significant challenge for the gridshell structure is the lack of standardized construction guidelines and practices. Therefore, further research is needed on the definition of instructions and the corresponding design standards [5]. Several breakages during the manufacturing process can occur due to the lack of standards. These beams are a critical case that needs more studies. ML methods can discover hidden information about the performance of gridshells by learning the effect of the damages. The choice of an appropriate material has a significant influence on manufacture possibilities. The remarkable properties of GFRP ensure easy manufacturing due to their elasticity. There are still some potentials to improve the efficiency of the material, and further study is required on the long-term creep-relaxation behavior of fiber-reinforced materials [5].

Another challenge in this case is joining the membrane to the grid nodes. Therefore, it is essential to estimate its effect on the gridshell structure for manufacturing process [5], [19]. Improvements are required regarding the assembly details of grid nodes and the membrane. One of the most critical problems about GFRP elastic gridshell structure is that it is not a permanent structure, and it can only be used as temporary structures. To solve this problem, it needs higher stiffness [62]. Therefore, design, form finding process, and choosing a suitable material requires further development to open new possibilities for manufacturing.

6.2.3. Design methodology and digital workflow

The gridshell structure has undergone many changes over the years with the digital revolution. However, new methods must be developed to speed up and streamline the design process. To maintain the use of advanced analysis for gridshells, the development of computational methods for modeling complex three-dimensional gridshell structures, applying ML algorithms in the design phase, and considering new techniques for form finding must be prioritized [87], [88].

The design of gridshells can be facilitated with the aid of form finding methods [89]. To save time during the design process, design tools need to be optimized. Specifically, determining the curve angles is a time-consuming process. To address this problem, using an analytical approach would potentially reduce the required time [22]. Furthermore, to accurately extrapolate membrane tensions and nodal stresses during the erection process, a more precise numerical model with digital finite element solver needs to be refined [74]. The data collected from such a platform would contribute to preparing an efficient and reliable dataset for use in ML models.

Additionally, a powerful digital program is necessary for realistic simulation of the stresses that can arise due to deformation loads based on real-world conditions. Moreover, ML algorithms are powerful tools for developing predictive models of structural behaviors [90], [91]. Consequently, the occurrence of breakages during construction can be minimized. During computer form finding, better results can be obtained by integrating tectonic design to develop an aesthetically pleasing shape and a functional structure [44], [92]. The alternative methodology for solving this problem is shown in Figure 13 [52].

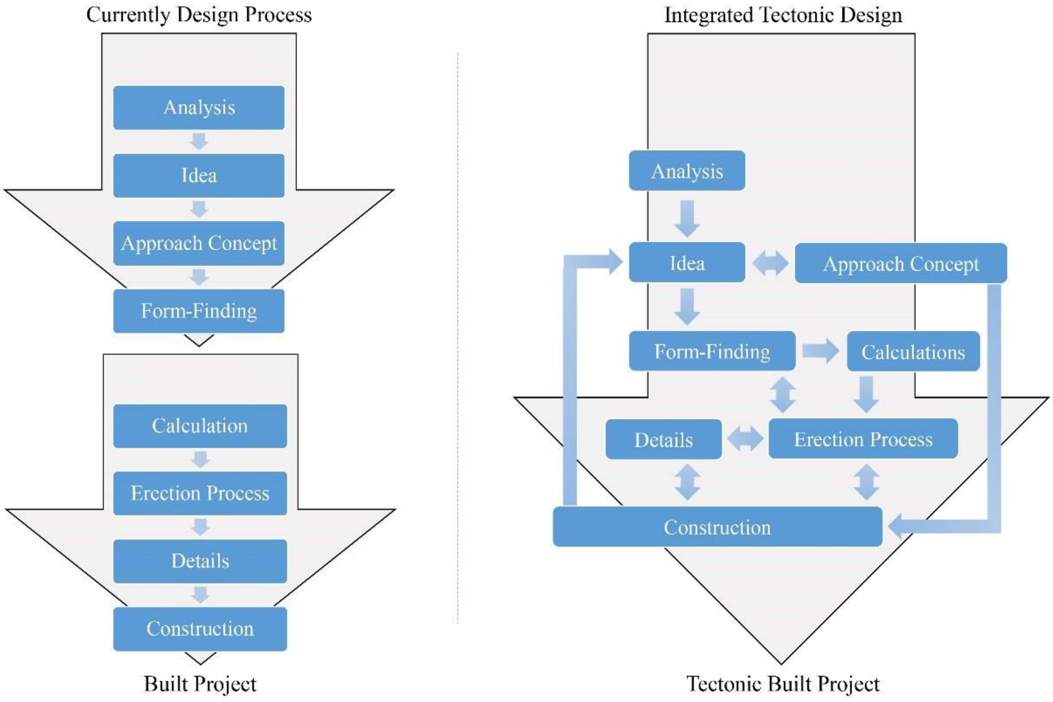


Figure 13. Design Process for elastic gridshells

The use of progressive digital methods is needed to be able to model complex three-dimensional gridshells, which include various design types, construction materials, connection details, and cost estimation. Furthermore, further developments are required in the field of computer form finding and analysis. This progressive digital form finding platform should encompass all the data that is relevant to the design and construction process of elastic gridshells.

6.2.4. Design and optimization of structures

Existing design methods will need to be updated for the optimization of GFRP elastic gridshell structures. The purpose of optimizing gridshells is to minimize stresses during the erection process and increase their resistance to external loading. Optimization leads to an increase in stiffness [93-96], and as a consequence, gridshells might be used as permanent structures. Optimization is required in the design and form finding process to decrease the curvature of the grid, optimizing mesh size and mesh pattern, optimize beams direction, avoid nodal connection weaknesses of continuous elements, design new forms of connections, and consider easier methods for bracing.

Also, the curvature of the grid profiles needs to be decreased for grid optimization [38], [41]. Variation in mesh size leads to minimum curvature, and future studies should focus on optimizing the mesh size and distribution of the profile [5]. Additionally, ML algorithms can be implemented for structural behavior prediction during the optimization process [90]. Thus, the required time for the optimization process is significantly reduced, and computational efficiency is enhanced. However, further studies on this subject are required.

Beam direction needs to be optimized during the design process to minimize stresses in the gridshell [25]. Currently, there are algorithms and software available that can generate the curvature of optimal profiles with a constant mesh size on the target surface [97]. By considering the effect of the grid pattern on the behavior of load bearing, other surface geometries must be explored.

Another critical limitation is the lack of options that can be used instead of membrane covering. To overcome this shortcoming, a novel concept consisting of a hybrid skin of a timber elastic gridshell structure braced with a concrete envelope have been proposed [7]. However, this solution can also be implemented to the GFRP elastic gridshells.

In the case of bracing, the optimization process can be done by integrating the bracing layer with the ability of elastically bent within the initial gridshell [41]. The weight of the swivel coupler is equal to 1.16 kg, which is a heavy element and represents about one third of the weight of the structure. This problem could be easily solved with dedicated design and optimization.

6.2.5. Temperature and moisture

The thermal comfort of the gridshell is far from ideal due to the poor thermal properties of the membrane. A forced-air heating system, located a few meters away from the structure, guarantees warmth during winter. The cost of energy remains relatively low; however, the solution is far from ideal due to the isolation shortages. The temperature increases very quickly during the day in the summer. The forced-air system is utilized for ventilation inside the structure, but the indoor temperature rapidly reaches 30°C, and as a result, the comfort level is insufficient. Therefore, one solution is to operate the mass in the period of the year with lower temperature. Considering economic reasons, cooling was impossible because the structure was only used a few hours a week. Thus, a better solution is required. Temperatures above 30°C and 50°C are problematic for people and the structure, respectively. The heat resistance of the membrane is about 70°C. At this temperature level, the interface layer experiences a decrease in resistance capacity for sliding, and the creep of the tubes speeds up [61].

Condensation is another problem in the winter. Often, droplets of water can damage wooden furniture. In the first months after completing the construction, this phenomenon is intense because the concrete slab is not completely dry. Therefore, one solution is to always maintain the temperature above 10°C inside the structure.

7. CONCLUSION

GFRP elastic gridshells are highly suited for temporary structures due to the minimal use of materials and their classification as sustainable structures. This paper explains elastic gridshell structures and introduces constructed buildings using this method. It mentions that GFRP is a suitable material for this type of structure due to its mechanical characteristics. The paper also reviews form finding and construction techniques based on existing research. It provides information about the building process of an existing structure, the Solidays gridshell, and explores opportunities and challenges in the use of gridshells.

This type of structure can create an exciting space by utilizing tubes and an economical and specific construction process for double-curved surfaces. Elastic gridshells composed of GFRP tubes enable the construction of highly optimized structural forms that would be time-consuming and expensive using traditional methods. Furthermore, they demonstrate advantages such as faster construction for curved surfaces, lightweight and high strength properties, and opportunities for deconstruction and recycling. Another advantage of elastic gridshell structures is that the deformation process induces the dominant stress, while additional loads like wind and snow have little effect on adding stresses. To minimize breakages during the construction process, the stresses due to deformation should be minimized.

This study shows that elastic gridshells are suitable for creating free-form structures. However, to increase their lifespan, better characterization of material behavior is necessary. Significant challenges remain in the construction techniques of elastic gridshells for creating free-form shapes. The construction process of GFRP elastic gridshells will need to be standardized further, and new rules for safety during and after construction must be implemented. Additionally, quality assurance is needed to ensure their reliability throughout their lifespan. Bracing, which can be time-consuming, is a crucial step that requires finding faster ways of stiffening. Moreover, a thin concrete surface can be used as an alternative to bracing and membrane.

Further research is expected to contribute to the construction of gridshells with broader spans. Additionally, the challenges of using these structures as permanent structures, rather than just temporary ones, remain unsolved. It may be possible to combine them with other techniques and use them as hybrid structures to address this problem. Currently, these structures, despite being lightweight and capable of covering large spans, have been seldom used in society due to the aforementioned problems. However, it is evident that after solving these issues, they have the potential to be widely used as both temporary and permanent structures. It is worth mentioning that as new methods are developed and their potential is better understood, the use of ML algorithms for GFRP elastic gridshells is expected to increase. Consequently, GFRP elastic gridshells offer potential and opportunities for constructing more complex double-curved surfaces by overcoming these challenges.

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CONFLICT OF INTEREST

The author (s) declared no potential conflicts of interests with respect to the authorship and/or publication of this paper.

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