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Statistical Analysis of b-value Parameter under Unconfined Uniaxial Compression Testing

Umair Ali Wani, Injila Hamid, Shahid Gulzar Wani *, Shafat Farooq

Department of Civil Engineering, National Institute of Technology, Srinagar, India-190006.

*Correspondence should be addressed to Umair Ali wani, Department of Civil Engineering, National Institute of Technology, Srinagar, India-190006. Iran. Tel: +916006593094; Fax: +916006593094; Email: waniumair786@gmail.com

ABSTRACT

In this paper, statistical analysis of the Acoustic Emission (AE) based *b*-value parameter has been carried out. In the initial phase, multi-linear regression was carried out on AE parameters released during the uniaxial compression of twelve cylindrical samples. The study exhibited that Counts, RMS values, and reciprocal of Absolute Energy are promising parameters to quantify the *b*-value. In a similar analysis, the AE behavior measured in terms of *b*-value was studied for standard cubes and cylinders. Firstly, the two-way ANOVA test was done on Counts, RMS values, and reciprocal of Absolute Energy, and a hypothesis was made that *b*-value measured in the said parameters would be the same. The results support the hypothesis and prove statistically that the three variables are the same no matter whether they are taken from cubic specimens or from cylindrical specimens, even though the cracking behavior of cubes and cylinders is different. The results are also verified by a one-tailed F-test.

Keywords: ***b*-value, multi-linear regression analysis, ANOVA test, F-test, Kolmogorov-Smirnov test, Lavene's test**

1. INTRODUCTION

The acoustic emission (AE) technique has invariably proved to be an efficient means of damage characterization tool without destroying the material conditions of concrete structures. The unique feature of AE is its operating principle which relies on the release of (AE) energy from within the material due to the various micro-structural changes generating elastic waves rather than being supplied externally. The AE technique evaluation method has been found to be more effective and accurate for developing damage grading systems [1]. Farhidzadeh et al. [2] studied that certain relevant features, such as the signal's maximum amplitude, energy, duration, and rise time, are extracted from the AE signals to identify the sources and to assess their significance which shows the source intensity. Shah et al. [3] concluded that variable AE amplitude is associated with the magnitude of fracture, and the b-value proves to be an effective parameter for studying the stages of fracture. Carpinteri et al. [4] tried to determine and characterize the damage growth using *the* *b*-value parameter, which shows progressive changes during the failure process while carrying out the uniaxial load tests on the specimens. Carpinteri et al. [4] defined b-value as the log-linear slope of the AE cumulative frequency magnitude distribution obtained using the frequency–magnitude distribution data by means of the Gutenberg-Richter relationship, which is generally used in seismology to characterize distributions of earthquake magnitude. This relationship is defined as:

$$\log_{10} N = a - bM.$$

Where, N is the incremental frequency (i.e. the number of events with amplitude greater than the threshold). Farhidzadeh et al. [2] slightly modified it by dividing the AE peak amplitude by a factor of 20; this is because the AE peak amplitude is measured in decibels, while the Richter magnitude of the earthquake is expressed in terms of the logarithm of maximum amplitude. Therefore, the modified equation is as follows:

$$\log_{10} N = a - b(A_{dB}/20),$$

where A_{dB} is the peak amplitude of AE event in decibel.

In the present study, statistical analysis has been carried out, which aims at finding a relationship between AE parameters and the b-value. The b-value parameter gives us information about crack movement and dominance of different types of cracks for any instant of time. The present study finds its significance by only using three parameters viz. Absolute Energy, Counts, and RMS to get an idea of crack propagation and crack pattern that is mainly studied using the b-value. The main objectives of this study are: (i) To find simple mathematical relationships between the b-value and AE parameters. (ii) To understand the AE behavior of cubes and cylinders while cracking. Shahidan et al. [1] studied the amplitude distribution of emission waves for the cracking process. It revealed that the trend of AE amplitudes in the b-value method managed to develop the process of micro and macro-cracking. Vidya Sagar [5] studied the importance of the b-value based on acoustic emission. This work also revealed the effect of loading rate and compressive strength of concrete on variation in AE-based b-value with the crack development in RC structures. Zhang et al. [6] performed a numerical study on cracking processes in limestone using the b-value. They found that the b-value of AE does not accurately evaluate the degree of damage in the rocks but can indicate the different states of damage during cracking processes. Zhang et al. [7] analyzed the rock burst tendency for granite and found that the variation in characteristics of the b-value of dry and saturated water granite is similar in the process of uniaxial compression. Zhou [8] studied the AE technique for concrete damage detection under loading and freeze-thaw cycles. Statistical analysis was also conducted to calibrate the parameters of Weibull distribution for damage probability density. Mukhopadhyay et al. [9] performed a statistical analysis of the assumed β- distribution of AE signals generated during the turning of a metal matrix composite. The statistical parameters used in this study were variance and coefficient of variation. Vidya Sagar et al. [10] performed a statistical analysis of acoustic emissions parameters and found that the Weibull distribution was a better fit than the Gaussian distribution for compressive strength and damage parameters. However, the Gaussian distribution was a better fit in the AE parameters. Main et al. [11] applied b-value analysis on AE data obtained from the beam. The trend of the b-value was compared with the development of the fracture process of the beam. A significant relationship was found between the trend of the b-value and the micro-cracking and macro-cracking that appeared during the test.

2. METHODOLOGY

2.1. MULTI-LINEAR REGRESSION

Regression analysis is a statistical tool to determine a relationship between a dependent variable and one or more independent variables. It can be used to find the strength of the relationship between these two types of variables and make future predictions about the topic under consideration. Regression analysis includes linear, multi-linear, and nonlinear analysis, which depends on the type of relationship between the dependent variable and the independent variable. If the dependent variable depends on more than one variable and the relationship is linear, then the analysis will be called multi-linear regression analysis.

Multi-linear Regression Analysis [12 - 15]: In multi-linear regression, any dependent variable 'y' is mathematically modeled as:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots\ldots + \beta_k x_k + \varepsilon.$$

In the above equation $x_1, x_2, \ldots, x_k$ are independent variables or predictors and 'y' is the output or the quantitative response of the model. $\beta_1, \beta_2, \ldots, \beta_k$ represent parameters of the coefficients of model and 'ε' is the error term. This 'ε' is the random variable whose expectation is equal to 0 and is having variance of σ^2. Here the linearity of model implies linearity of terms of model coefficients.

- **Advantages:** It is the simplest to implement and interpret. The relationship between the dependent and independent variables is the least complex.
- **Disadvantages:** It assumes a linear relationship between the variables, which is not, in general true for variables. It looks at the relationship of means between the dependent and independent variables, which is not the correct description of data as such a regression model can't provide an accurate relationship between the variables.

Estimation of Coefficients (Least square problem by matrix approach): The parameters of the model need to be estimated by using the sample data so as to fit the model. In this study, Excel and MATLAB software were used for analysis, but the matrix approach was used to get insight into the whole process.
Let us suppose, n experiments were conducted by using a k-tuple of inputs (which need not be distinct), and the responses obtained are as y_1, $y_{2,\dots,}$ y_n. The matrix representation of all the components can be given as:

$$\boldsymbol{X} = \begin{pmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{pmatrix}, \ \boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \quad \text{and}$$

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}.$$

The parameters of the model are given as:

$$\boldsymbol{\beta} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{pmatrix}$$

With the components set, the matrix representation of the model can be given as:

$$\boldsymbol{y} = \boldsymbol{X\beta} + \varepsilon.$$

In the present model $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ are independent variables having the same distribution as that of ε. If $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ are considered to be random variables, then $y_1, y_2, \dots, y_n$ are also considered to be random with any y_i depending only on ε_i.

The estimation of 'β' vector is required to fit the realized output vector, '**y**' and its expectation, '**Xβ**' so as to make further predictions. The most common method of estimation is the least sum of squares method.

Let $T(\beta1, \beta2, \dots, \beta k) = ||\mathbf{y} - \mathbf{X\beta}||^2 = \sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_1 - \cdots - \beta_k x_k)^2.$

The values of βi for which the above equation attains the minimum value are the parameter estimates, and the method of obtaining such estimates is known as the method of least squares.

Let the parameter estimates be given as:

$$\beta_0\hat{}=b_0,\ \beta_1\hat{}=b_1,\ \dots,\ \beta_k\hat{}=b_k.$$

Then the matrix representation of this estimate, **b** is given as:

$$\boldsymbol{\beta}^{\wedge} = \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{pmatrix}$$

The estimates for $b_0, b_1, \dots, b_k$ are obtained by setting the partial derivates of $T(\beta_0, \beta_1, \dots, \beta_k)$ with respect to the coefficients equal to zero and then solving the equations. Since the multilinear model formed is geometrically a hyper-plane, the equations obtained after partially deriving the model with respect to parameters are known as normal equations. The normal equations are given as:

$$\frac{\partial N}{\partial \beta_0} = -2\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_k x_{ik}),$$

$$\frac{\partial N}{\partial \beta_1} = -2\sum_{i=1}^{n} x_{i1}(y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_k x_{ik}),$$

$$\vdots \qquad \vdots \qquad \vdots \qquad \vdots$$

$$\frac{\partial N}{\partial \beta_k} = -2\sum_{i=1}^{n} x_{ik}(y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_k x_{ik}).$$

Setting the equations equal to zero, the system of equations can be simplified as:

$$\boldsymbol{X}^T(\boldsymbol{y} - \boldsymbol{Xb}) = \boldsymbol{0} \text{ or we have } \boldsymbol{X}^T\boldsymbol{Xb} = \boldsymbol{X}^T\boldsymbol{y}.$$

If $\mathbf{X^TX}$ is non-singular matrix, then **b** is given as:

$$\boldsymbol{b} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{y}.$$

This estimation is done by software as it involves a lot of calculation.

2.1.1. Properties

The value of **b** can also be written as:

$$\boldsymbol{b} = (\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T(\boldsymbol{X\beta} + \varepsilon).$$

Since the expectation of ε is zero, the expectation of b_i is given as:

$$E(b_i) = \beta_i.$$

The above equation implies parameter estimates are unbiased.

The matrix $C = (\boldsymbol{X}^T\boldsymbol{X})^{-1}$ is k+1 ordered matrix, which contains information about the variance and covariance. The two quantities are given as:

$$\mathrm{var}(b_i) = c_{ii}\sigma^2 \text{and } \mathrm{cov}(b_i, b_j) = c_{ij}\sigma^2.$$

Given the estimates, b_i, the response estimate is given as:

$$y_i\hat{} = b_0 + b_1x_{1k} + \dots + b_kx_{ik}.$$

The residual obtained from it is given as:

$$e_i = y_i - y_i^{\wedge}.$$

This residual is that response that is unexplainable to the model.

The quantity $\sum_{i=1}^{n} {e_i}^2 = \sum_{i=1}^{n} (y_i - {y_i}^{\wedge})^2$ is called sum of squares of errors and is denoted by SSE.

We know that E(ε_i)=0 and $\mathrm{var}(\varepsilon_i) = \sigma^2$. The expectation of SSE is given as:

$$E(SSE)=(n-k-1)\sigma^2.$$

Or we have, $E\left(\frac{SSE}{n-k-1}\right) = \sigma^2.$

The above equation is used to get an unbiased estimator for variance known as mean square error denoted by MSE and given as:

$$\frac{SSE}{n-k-1} = MSE.$$

We have SST=$\sum_{i=1}^{n} (y_i - \bar{y}_i)^2$ where $\bar{y}_i = \frac{\sum_{i=1}^{n} y_i}{n}$ and SST is total sum of squares.

Also, SSR=$\sum_{i=1}^{n} ({y_i}^{\wedge} - \bar{y}_i)^2$ where SSR denotes sum of squares of regressions.

The quantity, MSR known as mean square of regression is given as:

$$MSR = \frac{SSR}{k}.$$

Statistics of Regression: If the parameters of the regression model viz. $\beta_1, \beta_2, \ldots, \beta_k$ equal to zero, then it is interpreted that the regressors have no effect on the response. A similar interpretation can be associated with any predictor which has a coefficient equal to zero. When testing the significance, there is always a distribution associated with which we calculate probability. In fact, it is assumed that all the random variables, ε_i have N(0,σ^2) distribution.

While testing the significance of the model, the null hypothesis, H_0 is established as:

$$H_0: \beta_1 = \beta_2 = \ldots = \beta_k = 0.$$

Dthe alternate hypothesis, H_1 claims that at least one of the coefficients is not equal to zero. We can show that if null hypothesis is true, then the quantity,

$$F = \frac{MSR}{MSE},$$

is F-distributed with k and n-k-1 degrees of freedom.

If H_0 isn't rejected, the model would not be useful.

Assumptions of Multi-linear Regression:
The main assumptions considered for multi-linear regression analysis are enlisted as follows:

- There is a linear relationship between dependent and independent variables.
- The observations are independent of each other
- The residuals are normally distributed.
- The residuals have equal variance.

2.2. F-DISTRIBUTION and F-Test [13]

The comparison of standard deviations between two samples can be done with their sample variances and also by using the F-distribution. If any two independent random variables, V_1 and V_2 are χ^2 distributed with v_1 and v_2 degrees of freedom, respectively, a random variable:

$$F = \frac{\frac{V_1}{v_1}}{\frac{V_2}{v_2}},$$

is said to have F-distribution with v_1 and v_2 degrees of freedom. The probability density function of random variable, F is a Beta function of second type which has not been mentioned here because of its complicatedness.
Suppose S_1^2 and S_2^2 are the sample variances of two independent samples with size n_1 and n_2, respectively, such that the corresponding populations are normally distributed with standard deviations σ_1 and σ_2, then, the random variables:

$$V_1 = \frac{(n_1-1){S^2}_1}{{\sigma_1}^2} \; and \; V_2 = \frac{(n_2-1){S^2}_2}{{\sigma_2}^2},$$

are independent χ^2-distributed with n_1-1 and n_2-1 degrees of freedom. Thus, a random variable:

$$F = \frac{\frac{{S_1}^2}{{\sigma_1}^2}}{\frac{{S_2}^2}{{\sigma_2}^2}},$$

is F-distributed with n_1-1 and n_2-1 degrees of freedom.
For using the F-test, the null hypothesis, H_0 for any two populations A and B is established as:
H_0: The variance of population A= The variance of population B against the alternate hypothesis, H_1 that population variances are different.
As such, the f-statistic under the null hypothesis takes the form as:

$$F = \frac{{S_1}^2}{{S^2}_2}.$$

The significance level of the test is usually set to 0.05.
Given the null hypothesis H_0, the value of F is calculated either by using standard tables or by the software. The calculated F is compared with the F-critical value; if F falls in the critical region, the null hypothesis is rejected and if

the calculated F value is less than F- critical, the null hypothesis is accepted at a significance level of 0.05.

2.3. ANALYSIS OF VARIANCE (AVONA) TEST [14]

ANOVA is a statistical test to compare two or more groups of data. It has mainly two types.

One-way ANOVA: It compares the variance of means within a sample while considering a single independent variable. In other words, it establishes whether there exists any difference between three or more groups while comparing their means.

Two-way ANOVA: It is similar to one-way ANOVA. However, in this test, each sample is defined in two ways and, as such, put into two categorical groups.

Before doing the AVONA test, the authors have to go for checking the assumptions, the failure of which can produce wrong interpretations of data. The assumptions are given as follows:

Independence: It means each sample has been drawn independently and is independent of other samples.

Normality: It means that the sample has been taken from the population following a normal distribution.

Equality of Variance: It means that the variance of data in different groups is the same.

2.3.1. Hypothesis of two-way ANOVA

1. The null hypothesis for the k row population means is:

H₀: $\mu_{1.} = \mu_{2.} = \cdots = u_{k.}$.

- Which means that there is no difference between means of the rows.
- The alternate hypothesis is that at least one pair of means differ.

2. The null hypothesis for the m column means is:

H₀: $\mu_{.1} = \mu_{.2} = \cdots = \mu_{.m}$.

- The alternate hypothesis would be that at least one pair of column means differ.

3. The null hypothesis for km interaction is:

H₀: all $(\mu_{km} - \mu_{k.} - \mu_{.m} + \mu) = 0$.

- Which means there is no interaction between the factors or there is no difference in km cell means that can't be explained by the differences among the row means, column means, or both.
- The alternate hypothesis is that there is the interaction between independent variables.

2.4. MATERIALS AND TEST SPECIMENS

The cement used in this experimental study was 53 grade Ordinary Portland cement conforming to IS 4031 (Part 1): 1996, river sand, and tap water [16]. The sand used was standard sand conforming to Zone – II as per IS 383:1970 [17]. For concrete specimens, crushed coarse aggregate with a maximum size of 10 mm and 20 mm was used. In the present study, 12 cylindrical samples were cast in molds having dimensions 100mm x 200mm for studying the relation between b-value and various other parameters of AE. 6 samples were prepared in cylindrical molds of dimensions 150mm x 300mm and 6 samples prepared in cube molds having dimension 150mm x 150mm were used for the purpose of comparing b-values obtained in both the cases. The corresponding mixture details are given in Table 1 and Table 2, respectively.

Table 1. Mixture proportions of concrete samples were used (for a single specimen) to compare the b-value with various AE parameters. [12 cylinders with dimensions 100mm x 200mm]

	Density (per m^3)
Cement (kg)	343
Cement : Sand : Aggregate	1 : 2.03 : 1.78
W/C Ratio	0.45

Table 2. Mixture proportions of concrete samples used (for a single specimen)

	Density (per m^3)
Cement (kg)	422.07
Cement : Sand : Aggregate	1 : 1.75 : 2.8
W/C Ratio	0.45

- to compare b-value for standard cubes and cylinders with dimensions 150 mm × 150 mm and 150mm × 300mm respectively. [6 cube samples with dimensions 150mm x 150mm and 6 cylinders with dimensions150mmx 150mm]

2.5. EXPERIMENTAL SET-UP

The AE monitoring system used in this experimental study consisted of piezoelectric sensors, pre-amplifiers, and a data acquisition system (PAC, NJ, USA). This system allows the user to record AE waveforms and AE parameters such as count, hits, rise time, duration, counts, peak amplitude, energy, signal strength, absolute energy, average frequency, reverberatory frequency, and RMS.A single differential resonant type AE sensor (R6D), with a frequency range of 35 kHz-100 kHz) was used to record the generated AE. The sensor was mounted on the side of the cylindrical mold in the middle from the bottom. The sensor features a dual BNC connector with an integrated twin axial cable existing on the side. The signals were amplified (gain) using a pre-amplifier to 40 dB and fed directly to an eight-channel AE acquisition system. The software package AE^{WIN}SAMOS was used for the data analysis. AE hits with peak amplitude greater than 45 dB threshold were recorded. Silicon vacuum grease was used as a coupling between the sensor and the sample to ensure good contact and smooth transmission of AE signals from the sample to the sensor

2.6. UNIAXIAL COMPRESSION TEST OF HARDENED CONCRETE

The specimen samples were allowed to be set and, after 18 hours of hardening, kept in a curing tank. The specimens were taken out and left for drying before the compression test. The specimens were tested under unconfined uniaxial compression using a servo-controlled hydraulic testing machine having 1200 kN capacity using the constant displacement method as per ASTM C-39 standards, where the rate of loading was 0.0083 mm/s. Simultaneously, the released AE was recorded. The AE sensor (R6D) was mounted on the side of the specimen, and different AE parameters, namely AE hits, energy, duration, average frequency (AF), absolute energy, and signal strength, were recorded. The experimental setup is shown in Figure 1.

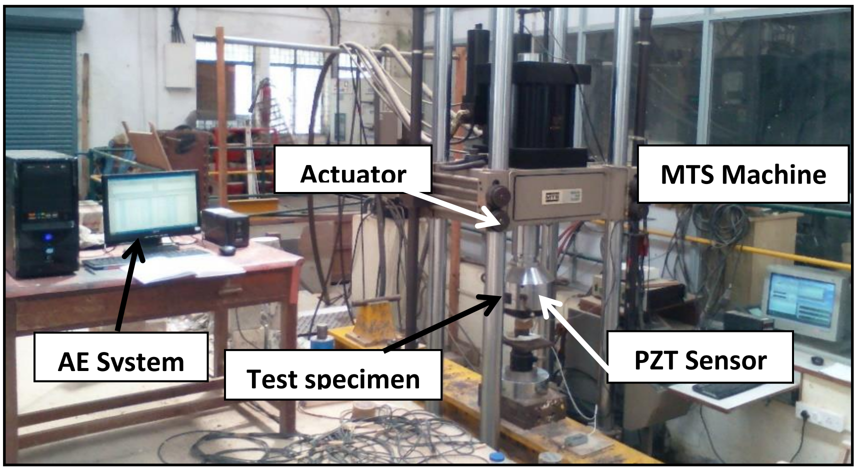

Figure 1. Experimental setup for testing hardened concrete, Structures Laboratory, Department of Civil Engineering, Indian Institute of Science, Bangalore

3. RESULTS AND DISCUSSION

AE data recorded for 12 cylinders of size 100mm×200mm which were subjected to uniaxial compressive loading is presented in Table 3.

Table 3. AE signal parameters corresponding to 100mm × 200mm cylinders

Specimens	Cumulative AE parameters recorded till failure			Parameter calculated
	Absolute Energy (atto joules)	**Counts**	**RMS(Volts)**	**b-value**
cyc1229	44734998543	2077988	726398	1.5233
cyc1329	29562632251	5661705	1265982	1.273
cyc1429	7854312796	2714842	1623643	2.2381
cyc3128	31030350867	4481966	968606	1.4597
cyc3228	4868414150	3115761	2517190	1.9104
cyc3328	10011956431	4103098	1441652	1.3544

cyc3428	17506711813	5674817	1065861	0.8396
cyc3528	1194039860	1611646	1548904	0.1086
cyc3628	8409539903	2115392	2095819	2.4724
cyc3728	11483332752	3832246	2250199	1.9829
cyc3828	4941504418	1320424	2309857	3.2985
cyc1129	16454756916	2923865	2296332	2.4945

In the first part of the results, multi-linear regression analysis has been carried out for the recorded AE parameters viz., Absolute energy, Counts, and RMS and the calculated parameter, namely b-value. A brief overview of the parameters used in the analysis is given: Absolute Energy: It is defined as the area under square voltage and time curve. The expression is given as:

$$\text{Absolute Energy} = \frac{1\sum_{\text{FTC}}^{\text{PDT}} V_i^{2}\Delta t}{R}.$$

Where FTC is the first threshold count, PDT is the peak definition time, and R is the resistance of the electric system (10kΩ). **Count:** For a given threshold, the count is defined as the number of times the signal crosses this threshold. **RMS:** It is the root mean square value of Voltage and is given as:

$$V^2{}_{rms} = \frac{\int V^2{}_i dt}{T}$$

Where T is the time period of the signal. ***b*-value:** It is the slope of the linear equation given as:

$$\log_{10} N = a - b(A_{dB}/20).$$

Where N is the number of AE hits with amplitude higher than A_{dB} and ‘a’ is a constant.

b-values were calculated and plotted using Excel and are resented in the Table 3 and Table 4.

Table 4. AE signal parameters corresponding to 150mm×300mm cylinders and 150mm ×150mm Cubes

		Cumulative AE parameters recorded till failure			Calculated parameter
		Absolute Energy (atto joules)	**Counts**	**RMS(Volts)**	**b-value**
cub1229	Cubes	2.05E+10	7313273	767672	1.0589
cub1329		1.04E+11	18375996	1227526	1.0921
cub1429		2.12E+10	18375996	1227526	1.182
cub1529		5.85E+10	10112059	1637806	1.1192
cub2129		1.87E+11	17463076	2557109	0.7706
cub2229		1.82E+10	6667096	2824838	2.2206
cycA1	Cylinders	6.02E+10	5325499	3957656	1.7637
cycA5		6.18E+10	3428350	780250	0.8751
cycA6		6.17E+10	4793722	3722731	1.852
cycA7		9.35E+10	5377012	3287712	1.7508
cycA9		7.17E+10	4696715	1695898	1.7637
cycA10		3.16E+10	4513669	2972494	1.715

Few plots showing b-values are shown below (Figure 2 – 5):

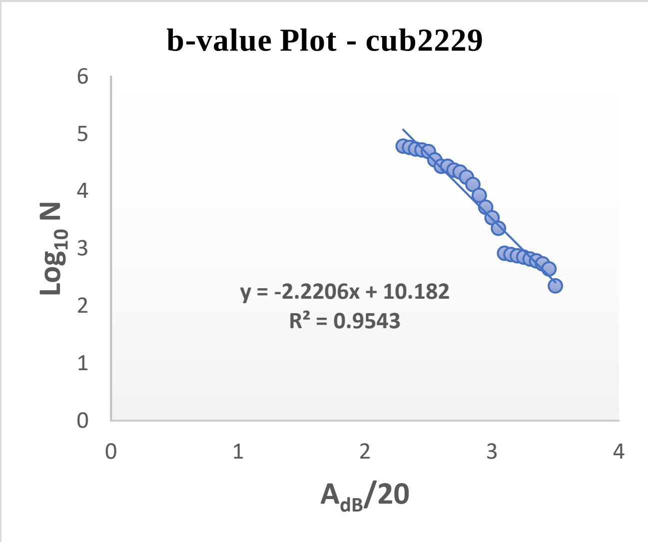

Figure 2. b-value Plot for cub2229

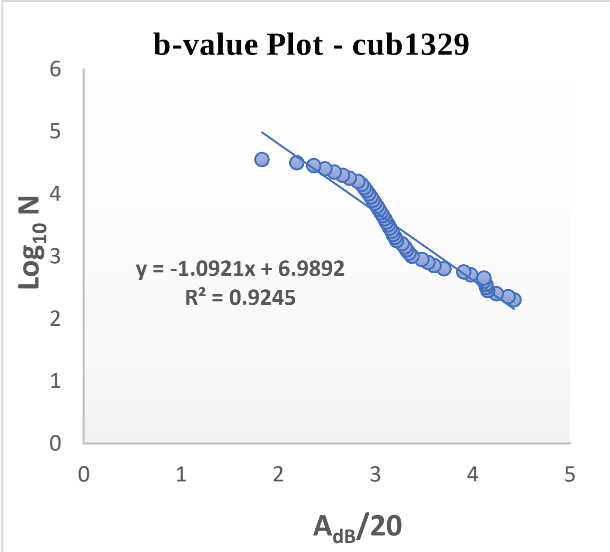

Figure 3. b-value Plot for cub1329

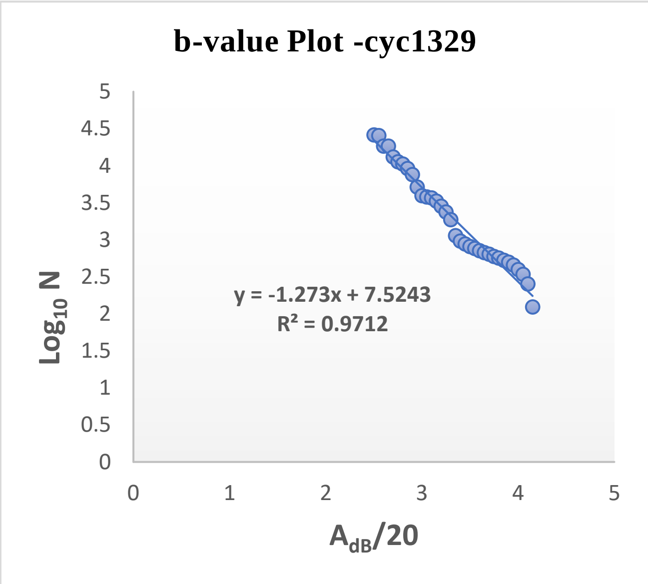

Figure 4. b-value Plot for cyc1329

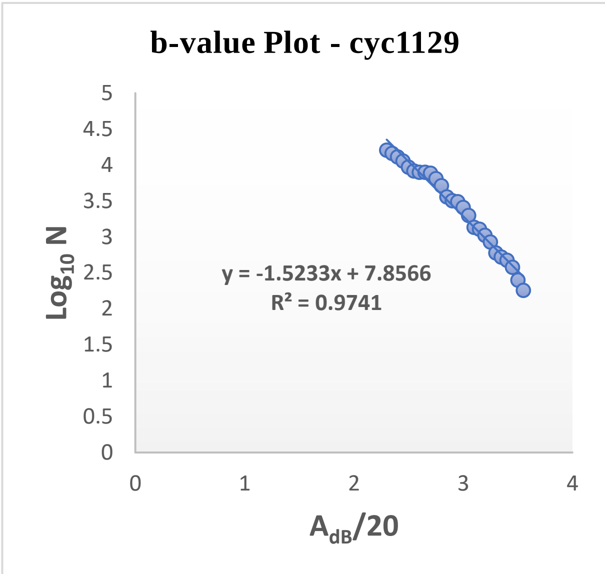

Figure 5. b-value Plot for cyc1229

Since the range of values for Absolute Energy, Counts, RMS, and b-values are different as such scaling of the values was done in the first stage so that all four parameters get the values on the same scale. The new values obtained were between 0 and 10. It is pertinent to mention here that scaling only changes the unit of the coefficient of regression. It doesn't influence the overall relationship. The multilinear regression analysis was carried out in Excel. The results of the analysis or the summary tables (Table 5, Table 6, and Table 7) are shown as:

Table 5. Regression statistics

Regression Statistic	
Multiple R	0.945098
R^2	0.893209
Adjusted R^2	0.853163
Standard Error	0.323396
Observations	12

Table 6. Regression Coefficients

	Coefficients	Standard Error	t-Statistic	p-value
Intercept	2.141594	0.508813	4.209	0.00296
Scaled RMS	0.70099	0.176122	3.980147	0.004061
Scaled Reciprocal of Absolute Energy	-0.30295	0.050119	-6.0447	0.000308
Scaled Counts	-0.33131	0.081989	-4.04089	0.003731

Table 7. AVONA table

ANOVA					
	Df (degrees of freedom)	**SS (sum of squares)**	**MS(mean square)**	**F**	**Significance F**
Regression	3	6.99808	2.332693	22.30434	0.000306
Residual	8	0.836678	0.104585		
Total	11	7.834758			

It was observed in the analysis that the Reciprocal of Absolut Energy, Counts, and RMS best fit the multi-linear model with the equation given as: $b = 2.141594 + 0.70099\text{RMS} - 0.30295\frac{1}{(\text{Absolute Energy})} - 0.33131\text{Counts}$.

The value of R^2 obtained in the analysis is 0.893209 which shows that 89.32% of the variation of b-value came from the regressors. Since R^2 is closer to one, it shows that he equation is a best fit.Null hypothesis for the multi-linear regression is given as: H_0: $\beta_1 = \beta_2 = \dots = \beta_k = 0$.
And the alternate hypothesis, H1, claims that at least one of the coefficients is not equal to zero. In the analysis part, the AVONA table gave an F-significant value of 0.000306, which is less than the critical significance of 0.05, and as such, we reject the null hypothesis, accepting the alternate hypothesis that at least one of the coefficients isn't equal to zero. In fact, the p-values for the regressors, reciprocal of Absolute Energy, Counts, and RMS are 0.000308, 0.003731, and 0.004061, respectively, all of which are less than 0.05. The values of 'p' less than 0.05 provide strong evidence against the null hypothesis; as such, these values indicate a strong relationship between the b-value and the said parameters. The results indicate that knowing the values of Absolut energy, Counts, and RMS values only, we can calculate the b-value parameter.
Hence, we can get information on cracks and crack patterns by knowing the values of these three parameters only. **Checking the assumptions: Normality:** The normality of residuals has been checked by plotting normal Q-Q plots of residuals. **Q-Q plots:** These plots act to check whether the data follows normal distribution or not. The data is arranged in ascending order and plotted against normal quantiles. It will match the normal quantiles, and the plot will follow a straight line if the normal distribution criterion is satisfied. The Q-Q plot of the residuals for the regression analysis is given in Figure 6.

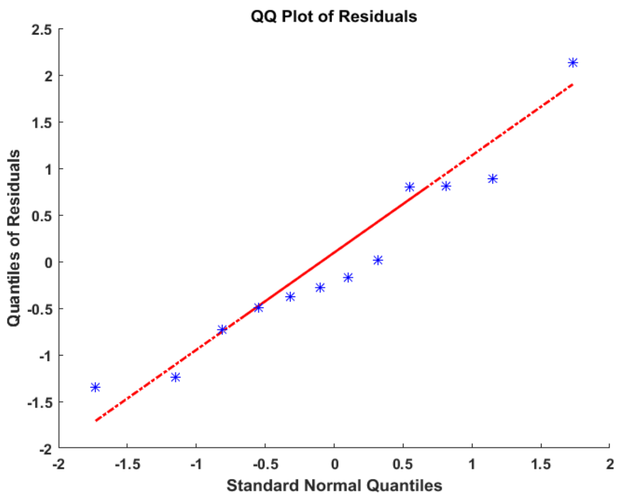

Figure 6. Q-Q plot of residuals

The plot shows residuals scattered about the straight line and as such, it can be concluded that residuals are normally distributed. **Independence:** The plots of residuals for the regressors are given in (Figure 7 – 9).

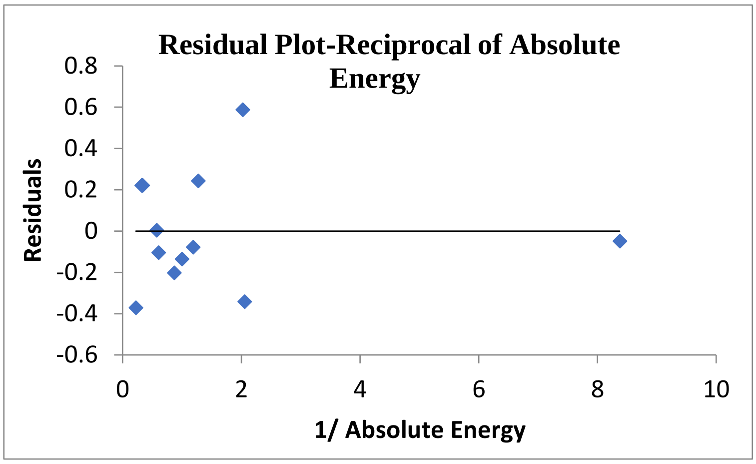

Figure 7. Plot of Residuals for Reciprocal of Absolute Energy

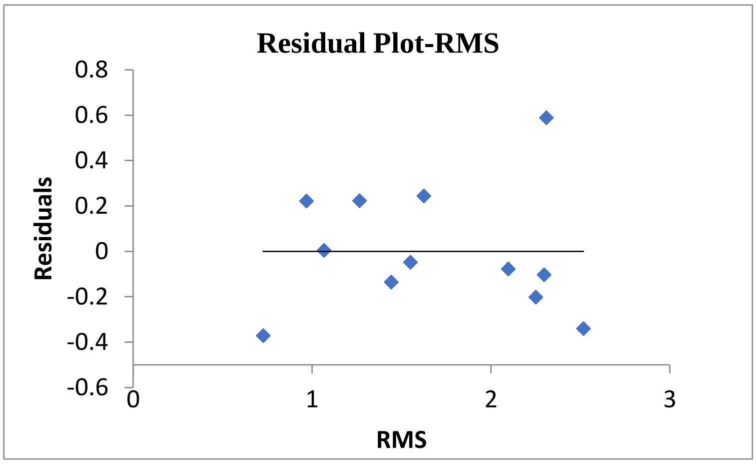

Figure 8. Plot of Residuals for RMS

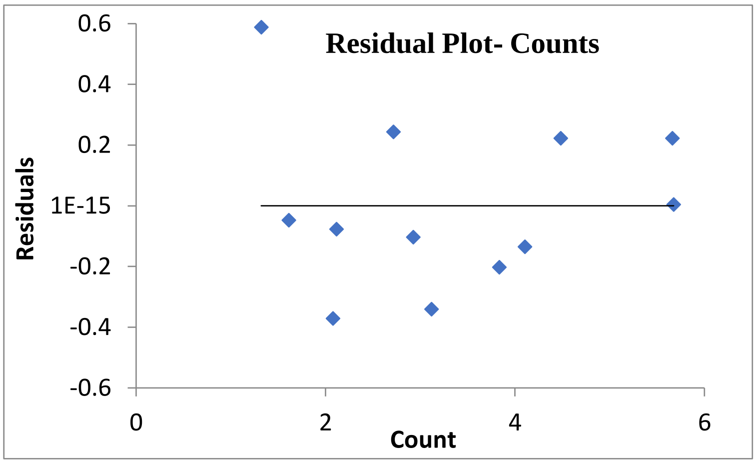

Figure 9. Plot of Residuals for Counts

Since all the points are randomly distributed throughout the plot, it clearly shows that the data points are independent of each other, as no pattern or dependence between the points can be established in the given plots.

Equal Variance of residuals: The plot of residuals for the regressors showed that no curve or a given pattern can be formed. Thus, verifying the assumption of equal variance of residuals.

Most influential regressor in the regression analysis:

The equation of fit as given by regression analysis is given as:

$$b = 2.141594 + 0.70099\text{RMS} - 0.30295\frac{1}{(\text{Absolute Energy})} - 0.33131\text{Counts}.$$

But this equation can't be used to decide which one of the three parameters is important for this linear function. One of the methods to decide the relative importance of regressors is by standardizing regression coefficients which can be done using Excel. The regressor, which has a maximum absolute value to its coefficient, is taken as the most important parameter. To determine the same, regressor coefficients were standardized, and the resulting coefficients are illustrated using a bar chart in Figure 10.

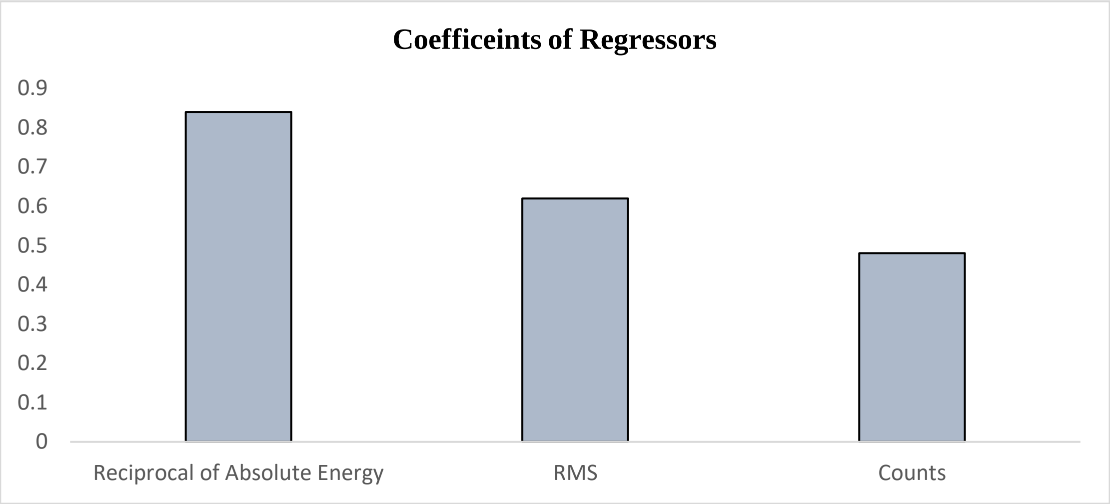

Figure 10. Bar chart for Standard coefficients

The bar graph show reciprocal of Absolute energy is the most important variable, followed by RMS and Counts. In this part of the results, AE behavior measured in the b-value of standard cubes (6 in number) and cylinders (6 in number) has been studied statistically. Since the b-value can be measured in terms of Absolute energy, Counts, and RMS, the two-way ANOVA for these independent variables can give us an idea about whether there is any difference in taking the samples from cubes or cylinders. The null hypothesis here will be established as: H0: There is no difference in AE behavior measured in terms of b-value against the alternate hypothesis, H1, that there is a significant difference between the variables taken either from cubes or cylinders. The reason for establishing such a null hypothesis is the different cracking behavior of cubes and cylinders [18]. Before establishing the actual results, the assumptions of two-way ANOVA have been verified: **Normality:** The normality has been established by plotting Q-Q plots and the Kolmogorov-Smirnov test. The Q-Q plots plotted in MATLAB are given in (Figure 11 – 19).

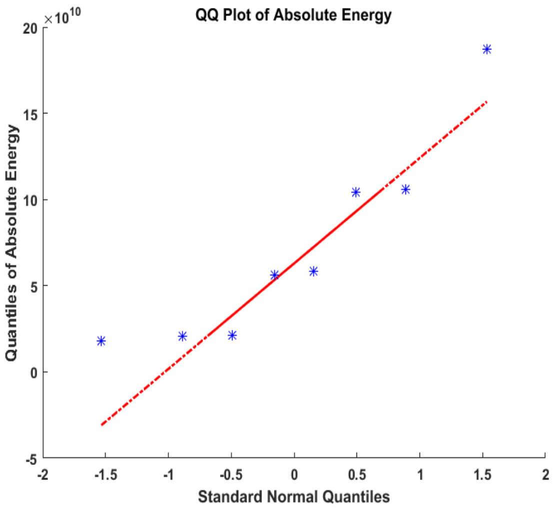

Figure 11. Q-Q plot of Absolute energy for cube samples.

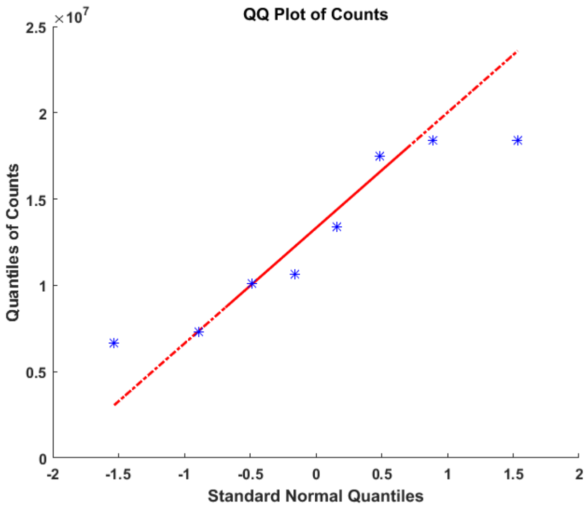

Figure 12. Q-Q plot of Counts for cube samples.

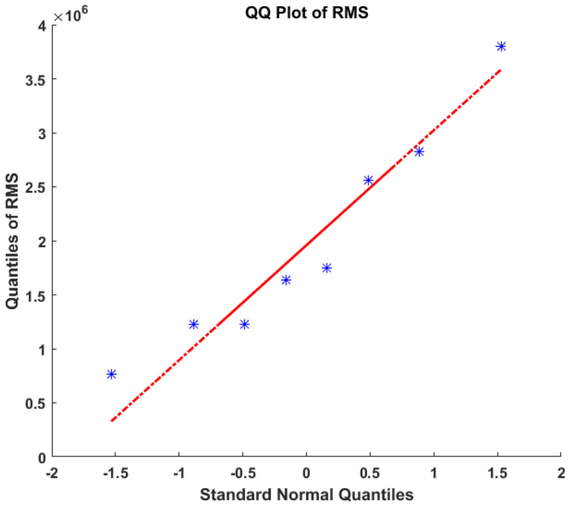

Figure 13. Q-Q Plot of RMS for cube samples.

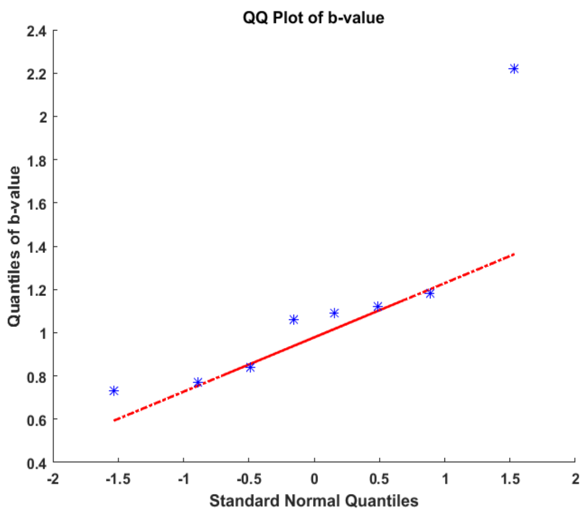

Figure 14. Q-Q plot of b-value for cube samples.

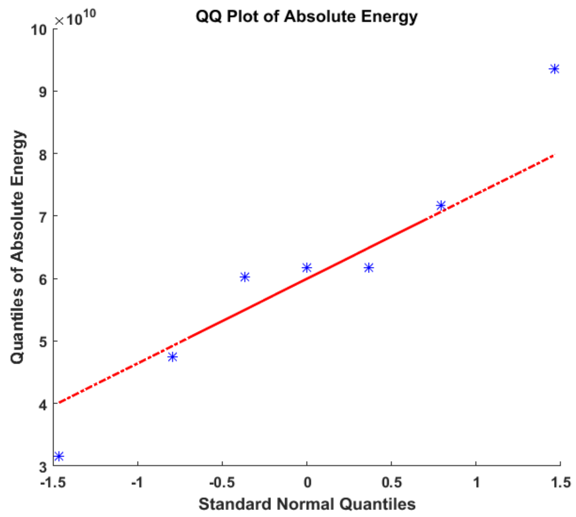

Figure 16. Q-Q Plot of Absolute Energy for cylinder samples.

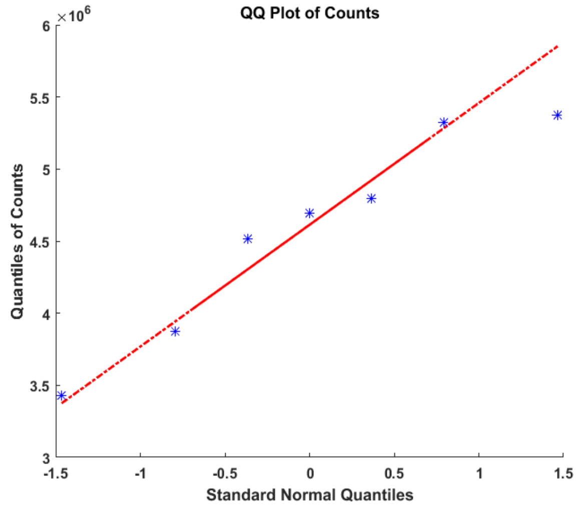

Figure 17. Q-Q Plot of Counts for cylinder samples.

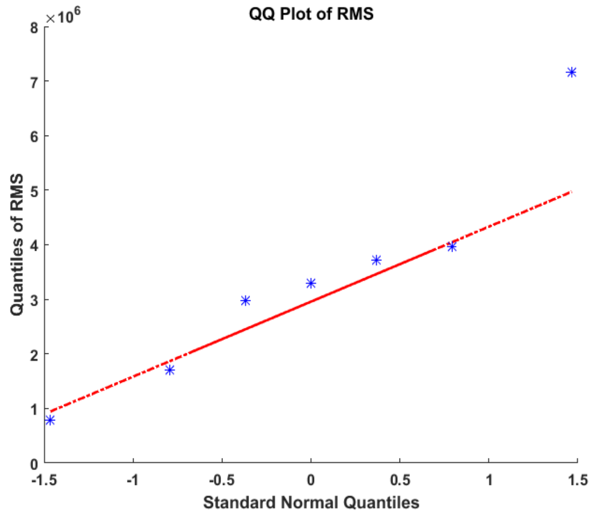

Figure 18. Q-Q Plot of RMS for cylinder samples.

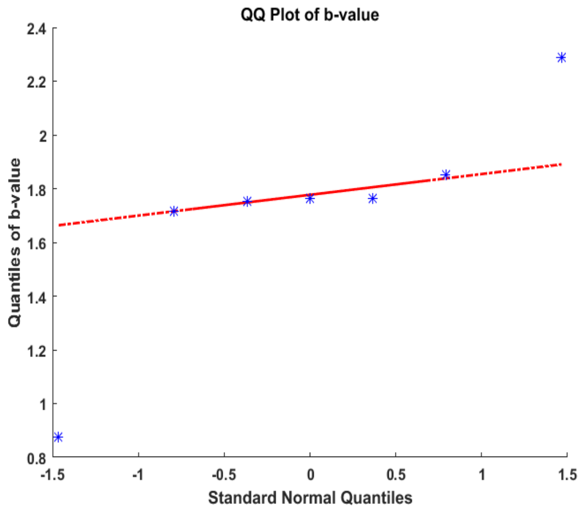

Figure 19. Q-Q plot of b-value for cylinder samples.

All the plots show that the variables follow an approximately straight line, and as such, it can be assumed that all the variables are normally distributed.

Kolmogorov- Smirnov Test: It is a rank test that checks whether a variable is distributed according to a known distribution. For each x, it calculates the difference between the percentage of the sample's data smaller than x and the probability of getting a value smaller than x from the known distribution. If the data follows a given distribution, these differences are very small. The static is taken as the maximum of all these differences. The K-S test data is shown in Table 8.

Table 8. K-S test table

Variables	Specimen	Max. difference	Critical values
Absolute Energy	**Cube**	0.79	0.51 for 6 samples
RMS		0.35	
Counts		0.28	
b-value		0.21	
Absolute Energy	**Cylinder**	0.27	0.51 for 6 samples
RMS		0.24	
Counts		0.23	
b-value		0.43	

It can be observed from the data that except for the Absolute Energy of cube specimens, all the variables have maximum difference less than the critical value, and as such, these variables are normally distributed. However, for the Absolute Energy of the cube, the Q-Q plot shows the straight-line trend; as such, we can't reject its normality. **Equality of Variance:** To check the equality of variance, Lavene's test has been used. **Lavene's Test:** This test is used to check the equality of variances between the two samples. This test is independent of the assumption of normality of data. The null hypothesis for this test is that the samples have equal variance. The test was done in Excel, and the results of the test are shown in the tables (Tables 9 and 10).

Table 9. Summary table

Summary				
Groups	**Count**	**Sum**	**Average**	**Variance**
Difference in Absolute Energy	12	3.854128	0.321177339	0.109940833
Difference in Counts	12	5.769261	0.480771767	0.084912688
Difference in RMS	12	11.98586	0.998821833	0.205216704

Table 10. AVONA table

Source of Variation	SS	df	MS	F	P-value	F critical
Between Groups	3.012193	2	1.506096629	11.29374195	0.000183383	3.284918
Within Groups	4.400772	33	0.133356742			

Since the F value is greater than F-critical as such we reject the null hypothesis that the samples have equal variance. In the experimental study, the sample sizes are equal, and the largest value of the difference of variance is only about four times the value of the smallest variance. Due to the robust nature of AVONA [19] for equal sample size, AVONA can be conducted even for the violation of this assumption. **Independence:** Since there is no test for independence, the data points are assumed to be independent as samples were randomly selected**.** The two-way ANOVA test was conducted in Excel, and the result tables are shown below: (Table 11, 12, 13)

Table 11. Data for cylinders

	Absolute Energy	Counts	RMS	Total
Count	6	6	6	18
Sum	4.091659	7.83075	10.24248	22.16489
Average	0.681943	1.305125	1.70708	1.231383
Variance	0.447959	0.316981	0.663833	0.60856

Table 12. Data for Cubes

	Absolute Energy	Counts	RMS	Total
Count	6	6	6	18
Sum	3.805152	2.813497	16.41674	23.03539
Average	0.634192	0.468916	2.736124	1.279744
Variance	0.040115	0.005019	1.546633	1.595892

Table 13. ANOVA table

Source of Variation	SS (sum of squares)	Df (degrees of freedom)	M(mean square)	F	p-value	F critical
Sample	0.021049	1	0.021049	0.041813	0.839359	4.170877
Columns	17.11266	2	8.556328	16.99629	1.16E-05	3.31583
Interaction	5.260321	2	2.630161	5.224551	0.011303	3.31583
Within	15.1027	30	0.503423			
Total	37.49673	35				

The above ANOVA table shows that for the samples of cube and cylinder, the F value is equal to 0.041813, which is less than the F-critical value of 4.170877, and thus we accept the null hypothesis that means the three established variables are the same for cube and cylinder specimens. It means that the three variables are the same, no matter whether they are taken from a cube specimen or from a cylinder specimen. This result can be interpreted as: The average values for Absolute Energy, Counts, and RMS are the same for both cubic and cylindrical samples. These results indicate that over the whole process of uniaxial compression testing, it can be concluded that these values would be the same for both cubic and cylindrical specimens. The table also shows that the F value for columns is 16.99629, which is greater than the F-critical value of 3.31583, so we reject the null hypothesis and accept the alternate hypothesis that means are different for all the three chosen variables. It can also be interpreted from the table that interaction effects are present i.e. interaction between independent variables impacts the output variable or dependent variable. This interaction is justified as multi-linear regression shows the dependence of the b-value on the reciprocal of absolute energy and not on the absolute energy variable, which has been used in two-way ANOVA tests. The two plots viz. Figure 20 and Figure 21 contain load-b-value-time for both cube and cylinder samples.

Figure 20. b-value-load-time for a cylinder.

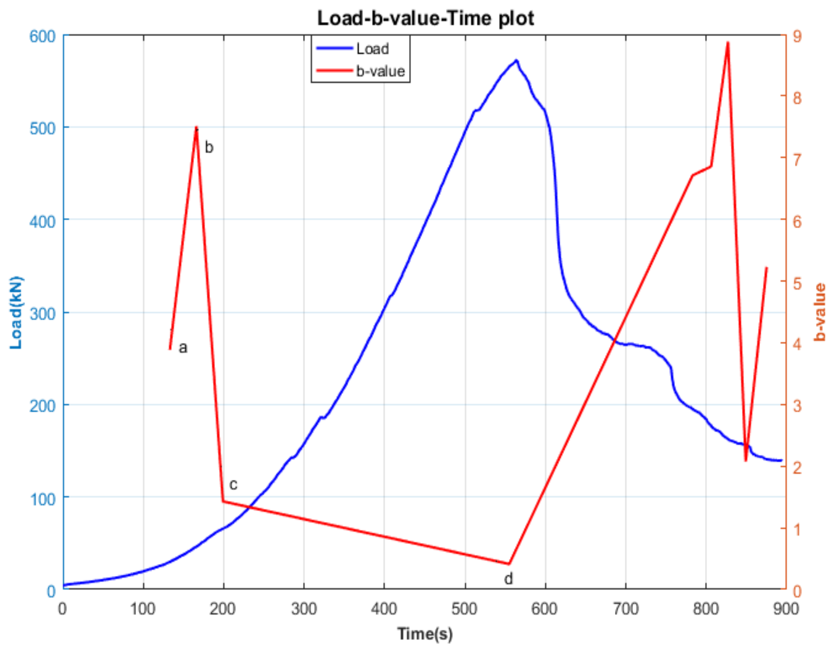

Figure 21. b-value-load-time for a cube.

The b-values were calculated after dividing the total number of hits into ten and twelve partitions. Those hits were then used to calculate b-values at different intervals of time. The plots show that the trend in the b-value in the cube and cylinder is the same when subjected to uniaxial compression. Since *the b*-value is the indicator of the crack growth and pattern in the concrete structures, it insinuates the fact that the crack pattern measured in terms of the b-value is the same for cube and cylindrical specimens. It can be seen in both plots that the b-value first increases from 'a' to 'b'. Then it decreases to 'c', which is a point before crossing the load-time curve. This b-value further decreases to 'd', which is a point below the peak loading in both plots. Again, in both, the plot's value of b-value increases after a point, 'd'. After crossing the load-time curve, the fluctuation pattern is almost similar in both curves, which proves the claim of AVONA testing. The three established variables are the measure of b-value; as such, we can make a valid claim that b-values at the end of the uniaxial compression test for cube and cylinder samples would be the same. To further support the claim, F-test has been done on the b-values. Table 14 shows the results.

Table 14. F-test table

F-Test Two-Sample for Variances		
	Cube b-value	**Cylinder b-value**
Mean	1.240566667	1.62005
Variance	0.250907811	0.135227
Observations	6	6
Df	5	5
F	1.855461907	
P(F<=f) one-tail	0.256958546	
F Critical one-tail	5.050329058	

The table gives an F value of 1.8555, which is less than the F-critical value, and as such, we accept the null hypothesis that the variances are the same for the two b-values obtained from cubes and cylinders. Since means and variances are the same, we can conclude that b-values obtained from cubes and cylinders are the same when these specimens are subjected to uniaxial unconfined compression.

4. CONCLUS ION

The b-value in any sample is not constant throughout the loading process. In fact, it fluctuates with time. The main conclusions drawn from this study are as follows: This study illustrates that the b-value can be estimated at any interval of time, provided data about Absolute Energy, Counts, and RMS up to that interval is given. It was also found that the reciprocal of Absolute Energy is more promising in estimating the b-value parameter as compared to Absolute Energy. As the study was done for the values obtained at the failure of samples, the regression analysis gave the p-values in favor of these respective variables as 0.00406, 0.00030, and 0.00373, all of which are less than 0.05, hence forming the best fit. The study depicted that the reciprocal of absolute Energy is the most important factor among the three identified variables. The statistical analysis done as ANOVA test and F-test also shows that the b-value pattern for cubic and cylindrical samples is almost the same or the average value of b-value for these two types of samples is the same. This shows that cracking behavior measured in terms of AE parameters is the same for both cubic and cylindrical specimens.

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AUTHORS CONTRIBUTION

This work was carried out in collaboration among all authors.

CONFLICT OF INTEREST

The author (s) declared no potential conflicts of interests with respect to the authorship and/or publication of this paper.

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