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Studying Performance of PVDs on Consolidation Behavior of soft Clayey Soils Using EFM, Mahshahr Oil Storages

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ABSTRACT

The main problem with saturated fine-grained soils is the slowness of consolidation procedure and the occurrence of large soil settlements. Generally, to expedite consolidation procedure, preloading method would be used along with prefabricated vertical drains. Soil improvement process under this condition would be subjected to design method and vertical drains' modeling. In the current paper and to understand the consolidation behavior of clayey soils improved with vertical drains, a parametric study has been performed by PLAXIS 2D finite element software. The results are indicative of an increase in average consolidation degree from 74 to 84% after a period of 6 months through the reduction of vertical drains' distances from 4.5 to 1.5m. Moreover, it became clear that an increase would be made in the rate of settlement, consolidation, and dissipation of excess pore water pressure through increase made in length and diameter of vertical drains and increase of their discharge capacity. Also, it became specified that increase of diameter is less effective on the expedition of consolidation procedure compared to that of distance reduction among drains.

Keywords: Soil improvement, Prefabricated vertical drain, Radial consolidation, PLAXIS 2D, Preloading

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1. Introduction

Population increase and urban sprawl, creation of new residential, as well as industrial and technological progresses have made an increase in humans' requirement to more optimum use of lands to construct roads, factories, installations, and etc. [1-3]. On the other hand, with consideration of a limited number of high-grade lands around cities and industrial zones, using alluvial lands with a low level of shear strength and high level of compressibility shows an increasing trend [4-6]. Nowadays, there are various methods of soil improvement including vibro-compaction, stone columns, deep soil mixing, deep dynamic compaction, explosive compaction, compaction grouting, sand compaction pile (SCP), micro piles, soil

replacement, soil improvement with lime and cement, electro-osmosis method, high-pressure injection, preloading and etc. The selection of optimum methods from among existing methods depends on various factors such as type of soil, depth of improvement required, costs, availability of facilities, and material required for the implementation of the concerned method, as well as previous experiments [7-10]. From among the above methods, preloading is one of the simple and economical methods to increase resistivity indicators of saturated fine-grained soils. From among the advantages of this method, reference could be made to the simplicity of implementation, measurement, and control of the ground

settlement, and in some cases, level of pore water pressures, through instrumentation and review of the method's behavior and performance during implementation. Practically, two types of preloading would be implemented through embankment construction and suction, along with vertical drains to increase the speed of consolidation settlements [11]. In general, soft clays need a long period of time for consolidation settlement, due to low permeability. Under this condition and to increase the speed of consolidation, prefabricated vertical drains are installed in soil. Vertical drains make drainage path short and cause an increase of consolidation process, and this in turn increases soil's capability in accepting new load through making an increase in bearing capacity [12]. Nowadays, many cases of research have been performed on the method so that soft soils would be improved. In 1925, sandy drainage technology was proposed by Moran to improve land in soft and deep deposits [10]. In Sweden (1952), the technology of using band-shaped prefabricated drains has been developed by Kjellman; and, each drain has been composed of a cardboard core and filter coverage. Filter coverage has been replaced with non-woven geotextile, later. Nowadays, prefabricated vertical drains (PVDs) are widely used for soil improvement [11]. Based on their analytical and numerical studies performed on the consolidation of clay soils, it was found by Indraratna et al. (2003) that reduction of saturation level of soil around vertical drain during installation in the initial consolidation phase can reduce suppression speed of excess pore water pressure. Also, as shown by them, specifying soil parameters near the drain is easier than the inside and outside undisturbed area around it [13]. An analytical solution to model consolidation with the help of vertical drains has been presented by Basu et al. (2006) through the introduction of five standard profiles. After computing the consolidation rate related to each profile, it was found that the occurrence of each kind of change in hydraulic conductivity of the undisturbed area would have resulted in a change of consolidation rate [4]. In 2008, consolidation was modeled numerically and theoretically, where the vertical drain has been available under the circular embankment. Comparing one dimensional consolidation degree of unit cell model, an equivalent value has been obtained by them for the coefficient of permeability of soil; and, the analytical solution of the continuity equation of radial drainage on column drainage walls has been provided by them [14]. In 2010, the behavior of marine clays under excess pressure with and without prefabricated vertical drain has been modeled by Bo and Arul Rajah, through PLAXIS finite element software; and, it was found that symmetric single-cell analysis and full-scale analysis of prefabricated vertical drains have well consistency with each other and with the results obtained from instrumentation [15]. In 2011, the effects of disturbed region's properties on the time required for preloading had been studied by Parsa Pajouh et al. through finite-difference software (FLAC); and, it was found that change of parameters in the disturbed region can considerably affect the duration of consolidation. It was also shown that parameters of the disturbed region such as size and permeability coefficient have a considerable effect on designing the procedure of preloading [16]. According to the comparison made on normal drains, and prefabricated suction drains performed by Bergado et al. (2014), it was

found that increase in settlement speed made by prefabricated suction drains is about 6 to 7 times higher than that of normal drains, and less disturbance is created in the soil around the drain. Using this type of drains can create fewer disturbances in the soil around the drain [17]. In 2015, conformity of 2D and 3D plain strain models for vertical drain systems was confirmed and proved by Yildiz [18]. Lu et al. (2016) extended the non-linear radial consolidation solution of Indraratna et al. (2003) to analyze the rate of consolidation of PVD-improved ground subjected to multi-stage loading and preloading-unloading-reloading schemes. Lu et al. (2016) included multiple PVDs inside a unit cell and developed analytical solutions for four loading schemes: (1) instantaneous loading, (2) ramp loading, (3) multi-stage instantaneous loading, and (4) multi-stage ramp loading, considering the effects of soil disturbance, i.e., constant horizontal hydraulic conductivity in the disturbed zone, and well resistance of all the PVDs [19]. Bo et al. (2016) examined the effectiveness of non-traditional drain installation patterns, such as circular drain ring and parallel drain wall patterns, versus conventional (square or triangular) drain installation patterns, on the rate of soil consolidation. The time required to achieve 90% degree of consolidation at 1 m drain spacing was 14, 60, and 250 days for drain rings, drain walls, and the conventional square pattern, respectively [20]. In this regard, the use of the vacuum consolidation technique in conjunction with surcharge preloading and PVDs, can enhance the efficiency of ground improvement works [21-22]. The effective depth of PVDs in soft clay soils has been modeled, taking advantage of the results obtained from instrumentation. The results from parametric studies showed that an increase of drains' depths would be resulted, in increase of project cost, with no increase in soil consolidation level [23]. Oliveira et al. (2015) compared three types of FE simulations of PVDs installed in Portuguese soft soil: two 2D plane strain simulations and one 3D simulation. The plane strain simulations were performed by (a) using an equivalent hydraulic conductivity k_{eq} for PVD-treated soil, and (b) replacing the axisymmetric flow around the PVDs by flow into equivalent drainage walls (simulated by permeable vertical lines). In the 3D simulation, the PVDs were modeled as equivalent square drains using highly permeable FE elements. The 2D simulations provided similar results to the computationally intensive 3D simulations with respect to embankment settlement and lateral deformation of subsoil. However, the excess pore pressures were found to be sensitive to the type of analysis performed, because of the distinct flow conditions imposed in each case. The 3D analyses predicted similar excess pore pressures to those obtained from (a) the 2D analyses, and (b) the field data; however, the 2D simulations with equivalent drainage walls did not predict the excess pore pressures accurately [24]. Indraratna et al. (2016) proposed an analytical solution for soil consolidation, considering the degradation of natural fiber drains over time. An exponential form of a reduction in drain discharge capacity was incorporated in the analysis. The dissipation of excess pore pressure got delayed significantly due to drain degradation, which depended on the magnitude of the decay coefficient [25]. According to Zhou and Chai (2017), most PVDs have a filter hydraulic conductivity greater than 10⁻⁴ m/s, which is much higher than the horizontal hydraulic conductivity of soft clay into

which PVDs are installed [26]. Indraratna (2017) presented an overview of recent theoretical and practical developments on soft ground improvement using PVDs with surcharge and vacuum preloading [27]. In the present paper, the behavior of soil improvement by PVD has been primarily analyzed numerically; and, the results have been

compared to those obtained from instrumentation. For better recognition of effective factors, a parametric study by PLAXIS 2D finite element software in axisymmetric condition has been performed on those variables having the effect of PVD design, such as installation distance, equivalent diameter, discharge capacity, and its length.

2. MATERIALS AND METHODS

To specify the correctness of computations, the output of finite element software has been primarily compared with the results obtained from the case study. In case of consistency, then software outputs could be used in the parametric study as valid data.

2.1. PROJECT DESCRIPTION

Mahshahr Oil Storages with 50 hectares area in Bandar-e Mahshahr has been established for oil storage (Figure 1). In

the region and due to the existence of soft clay subsurface layers and high groundwater levels, mainly settlements are from consolidation type because of the load imposed by oil storage tanks. In the project and to improved soil under the oil storage tank foundation, preloading with the help of embankment along with band-shaped drains with triangle pattern has been used with 1.5m in distance and 22m in depth [28].

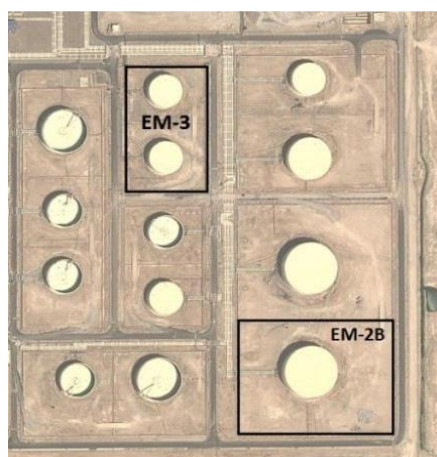


Figure 1. Layout of oil storage tanks (Mahshahr) [28]

The fuel storage tank under study has been tank EM-2B. To improve the soil under this tank through preloading, an

embankment with 14.66m in height and 9550m² in area has been used, according to the Figure 2.



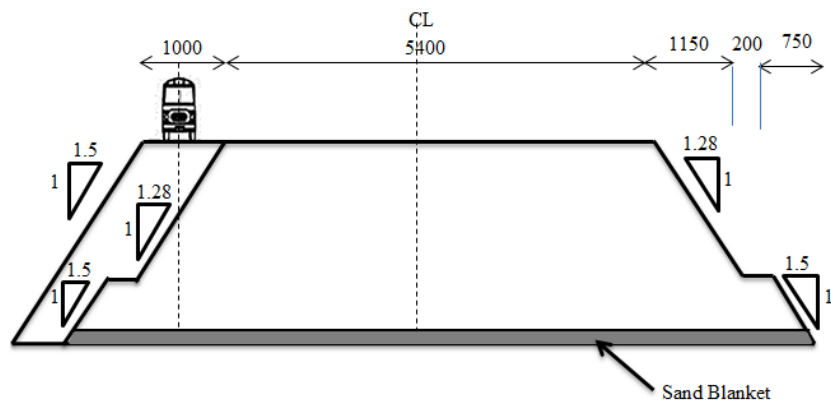


Figure 2. A view of embankment EM-2B [28]

Radial drains have been used from band-shaped Colbond drain CX1000 type (10×0.36 cm). According to Figure 3, they have been installed in the project site, in triangle array

[11]. Considering geotechnical studies performed in the site, geotechnical specifications, and soil layers in the project site are presented in table (1):

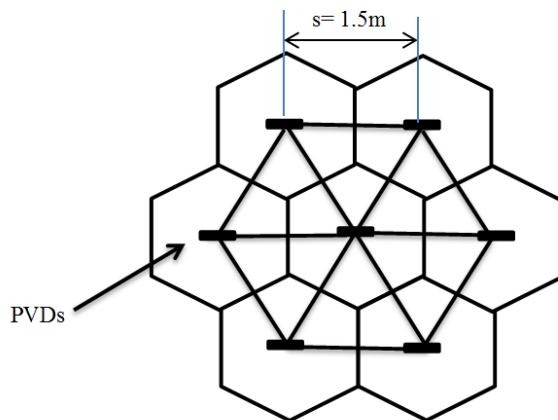


Figure 3. Drains' array in plan

Table 1. Engineering parameters of improved layers [28]

Thickness (m)	Soil type	Cc	Cs	γ_{sat} (kN/m ³)	γ (kN/m ³)	e _o	k _h /k _v	k _h (m/day)
10-0	CL	0.17	0.03	20.59	20.16	0.7	2	0.0086
22-10	CL	0.17	0.03	20.59	20.16	0.7	2	0.0086
38-22	CL	0.16	0.03	20.59	20.64	0.75	2	0.0086
48-38	CL	0.16	0.03	20.59	20.64	0.57	2	0.0086
73-48	CL	0.16	0.03	20.59	20.64	0.57	2	0.0086

2.2. VERIFICATION OF RESULTS

In a numerical analysis for every geotechnical problem, there are some specified stages observation of which in the analysis is a must. These stages are of similar principles in all software packages used with numerical methods in solving geotechnical problems, including defining problem's geometry, applying boundary conditions, meshing, and problem-solving. In the present study, all cases of modeling have been performed through PLAXIS

2D finite element software. For numerical analysis and meshing, axisymmetric condition and 15 triangular node elements have been used, respectively; and, boundary conditions have been modeled in a way that drainage in the lowest layer of soil would be unilateral. That is, drainage would not be done from the layer under the clay layer. [Figure 4](#) shows embankment modeling in finite element software.

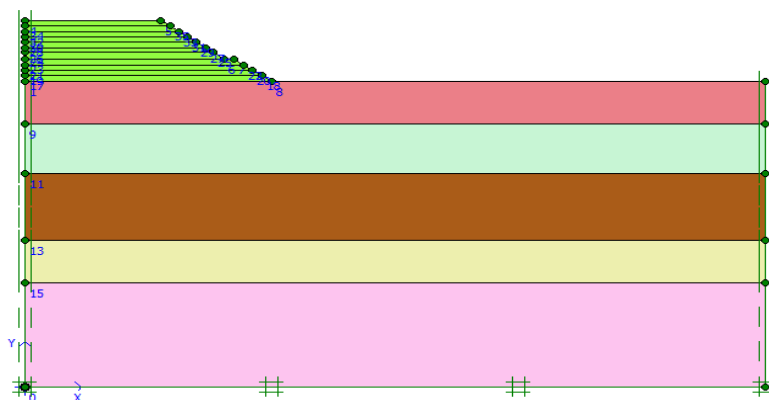


Figure 4. Modeling by PLAXIS 2D

Required parameters to model soil disturbance and good PVD resistivity through the Kve method are presented in

[table \(2\)](#).

Table 2. Drains' properties [27]

$d_s(\text{cm})$	$d_w(\text{cm})$	n	s	$L(\text{m})$	$q_w(\text{m}^3/\text{year})$	μ	$k_{ve} (\text{m}/\text{day})$	k_v	k_h
28.49	5.18	30.38	5.5	22	140	28.8	0.15	0.0043	0.0086

It has to be noted that, Mohr-Coulomb and Soft Soil Creep constitutive models have been selected for embankment and clay soils, respectively [29]. Stage construction of the embankment EM-2B is shown in [Figure 5](#) the settlement

value obtained from finite element analysis has been compared with the results from instrumentation in [Figure 6](#).

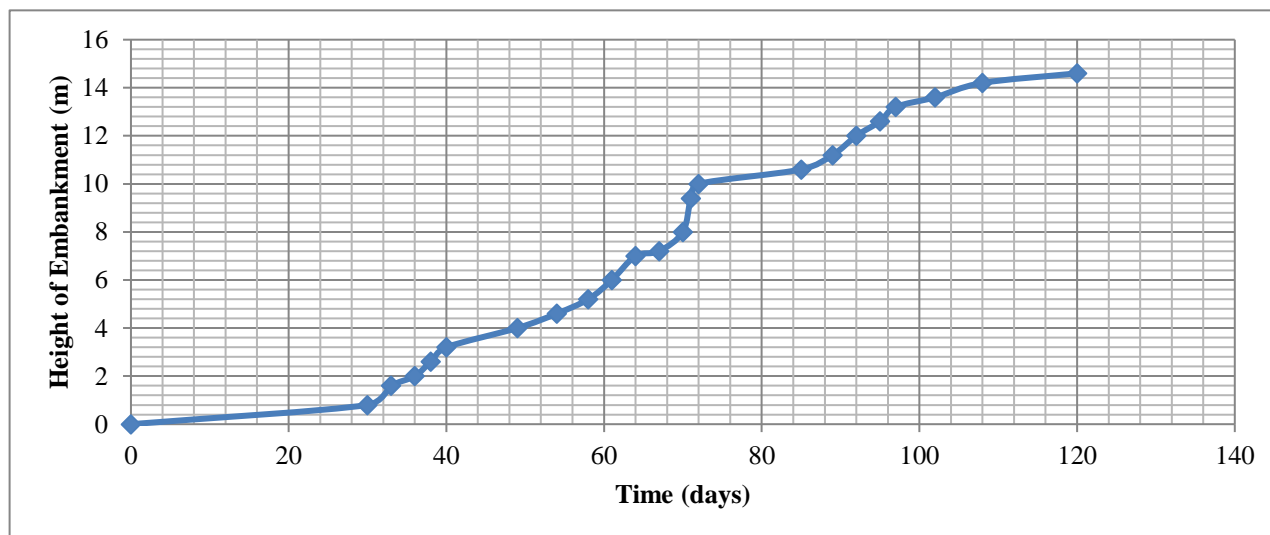


Figure 5. Stage construction of embankment EM-2B

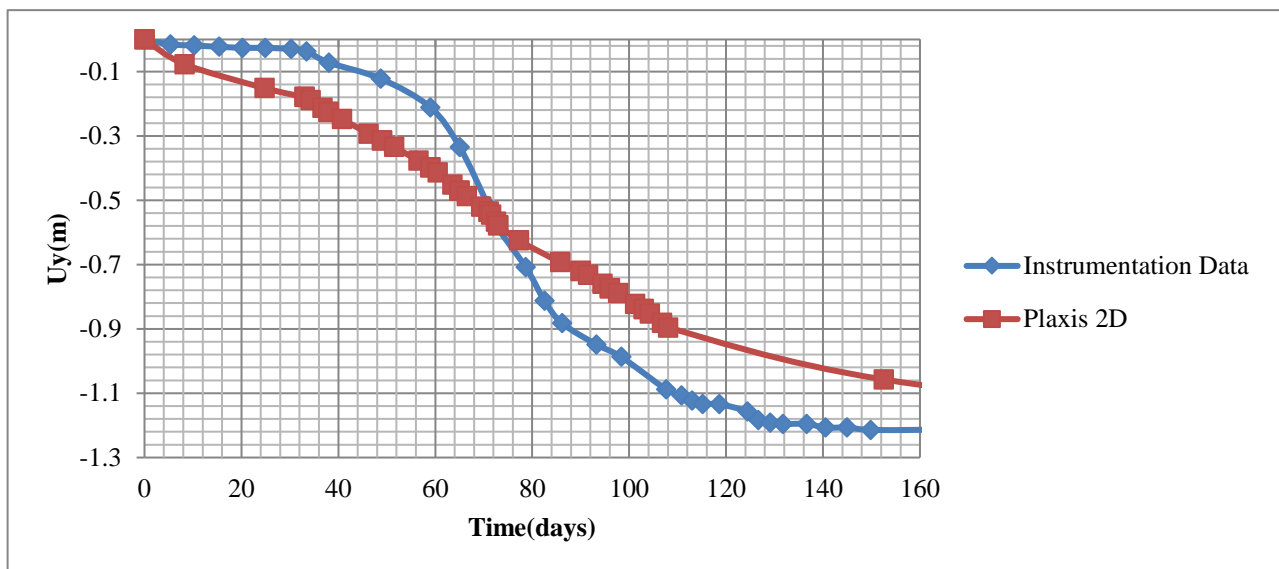


Figure 6. Settlementure value obtained from numerical solution and instrumentation

According to [Figure 6](#), there is a good match between settlement computed through numerical analysis in a 160 days’ time interval (1.27m) and the instrumentation results (1.15m). Therefore, software outputs could be used as valid

data in future parametric studies. It has to be remembered that little difference existing between the results from numerical analysis and those of instrumentation would have resulted in high-level design.

3. RESULTS AND DISCUSSION

After comparing the results from numerical modeling and instrumentation, a series of parametric studies have been performed on PVD design variables; and, effects of changing different design parameters such as installation distance, diameter, length, and discharge capacity on the

behavior of the land improved by PVD has been studied based on changes in consolidation rate, settlement rate, and dissipation rate of excess pore water pressure. The numerical variables used in the study are presented in [table 3](#).

Table 3. Changes of installation distance, diameter, discharge capacity, and PVDs’ lengths

Changes in installation distance				
S(m)	L(m)	d _w (cm)	q _w (m ³ /year)	k _{ve} (m/day)
1.5	22	5.18	140	0.15
3	22	5.18	140	0.0398
4.5	22	5.18	140	0.0199
Changes in diameter				
S(m)	L(m)	d _w (cm)	q _w (m ³ /year)	k _{ve} (m/day)
1.5	22	5.18	140	0.15
1.5	22	10.36	140	0.161
Changes in discharge capacity				
S(m)	L(m)	d _w (cm)	q _w (m ³ /year)	k _{ve} (m/day)
1.5	22	5.18	140	0.15
1.5	22	5.18	420	0.312

1.5	22	5.18	1260	0.492
Changes in length				
S(m)	L(m)	d _w (cm)	q _w (m ³ /year)	k _{ve} (m/day)
1.5	11	5.18	140	0.0935
1.5	22	5.18	140	0.15
1.5	44	5.18	140	0.177

3.1. DISTANCE VARIATION

Supposing diameter to be fixed, length and discharge capacity of PVD, curves of average consolidation degree, settlement, and dissipation of excess pore water pressure of

land improved by PVD for a different distance of 1.5, 3, and 4.5m are shown in [Figures 7, 8, and 9](#).

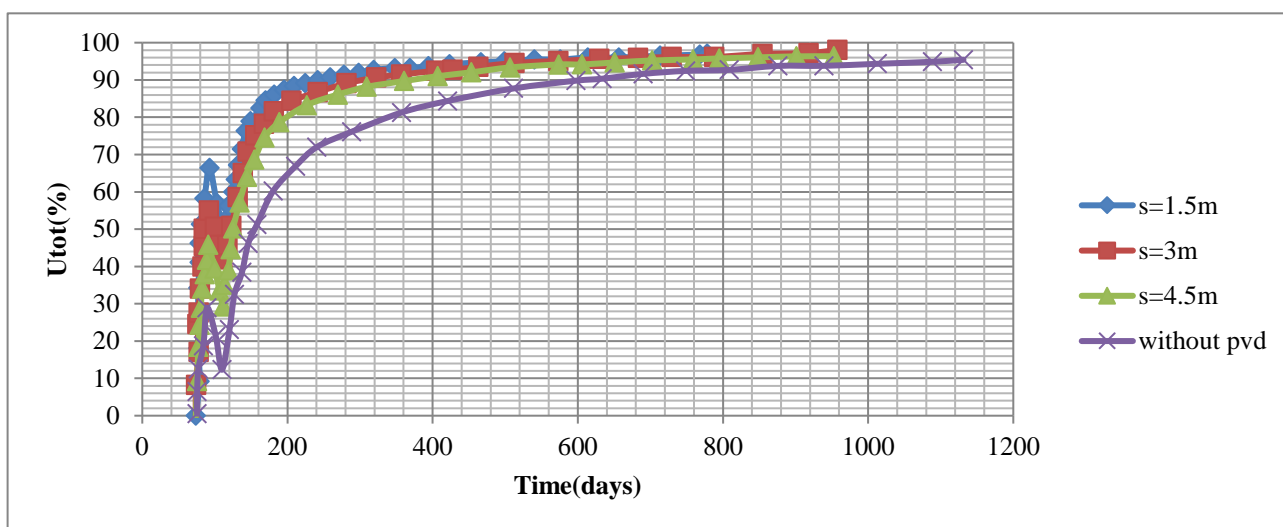


Figure 7. Comparing consolidation rate of the land improved by PVD for different distance of 1.5, 3, and 4.5m

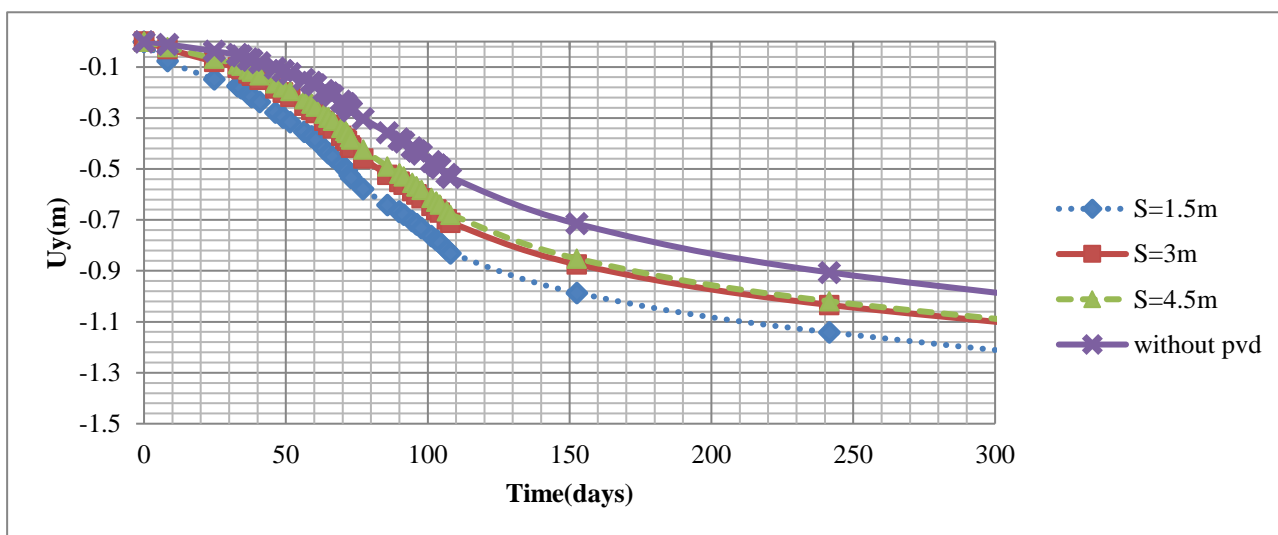


Figure 8. Comparing settlement of the land improved by PVD for different distance of 1.5, 3, and 4.5m

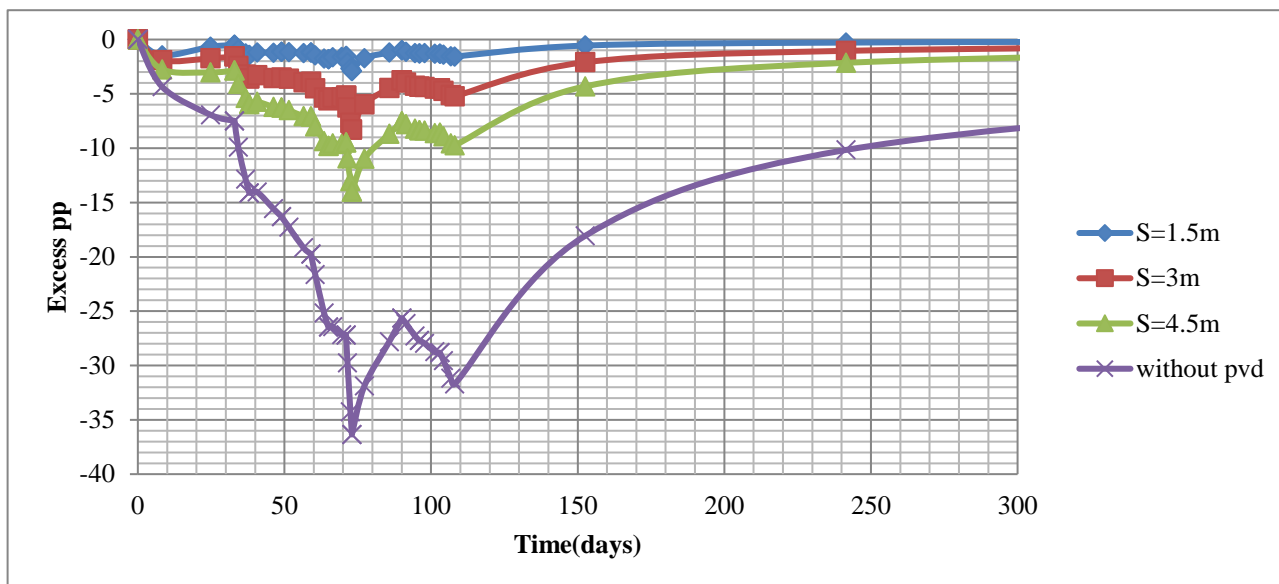


Figure 9. Comparing dissipation of pore water pressure for different distance of 1.5, 3, and 4.5m

As expected, the decrease in the distance between the PVDs increases their number per unit area, which increases the dissipation rate of the excess pore water pressure and thus increases the rate of the consolidation. The obtained results showed that the closer would be distances between PVDs, the more consolidation speed would increase. Also, more settlement occurs, and excess pore water pressure would be

more quickly suppressed. For example, when the distance between PVDs reduces from 4.5 to 1.5m, average consolidation degree U (%) for a period of 6 months has been increased from 74 to 84%; whereas, with no PVD installed, consolidation rate for the aforementioned period has been equal to 56.83%.

3.2. Diameter variation

Also, with consideration of installation distances to be fixed, length and discharge capacity, curves of average consolidation degree, settlement, and dissipation of pore

water pressure of the land improved by PVD for two different diameters (according to table 3) of 5.18 and 10.36cm are shown in [Figures 10 to 12](#).

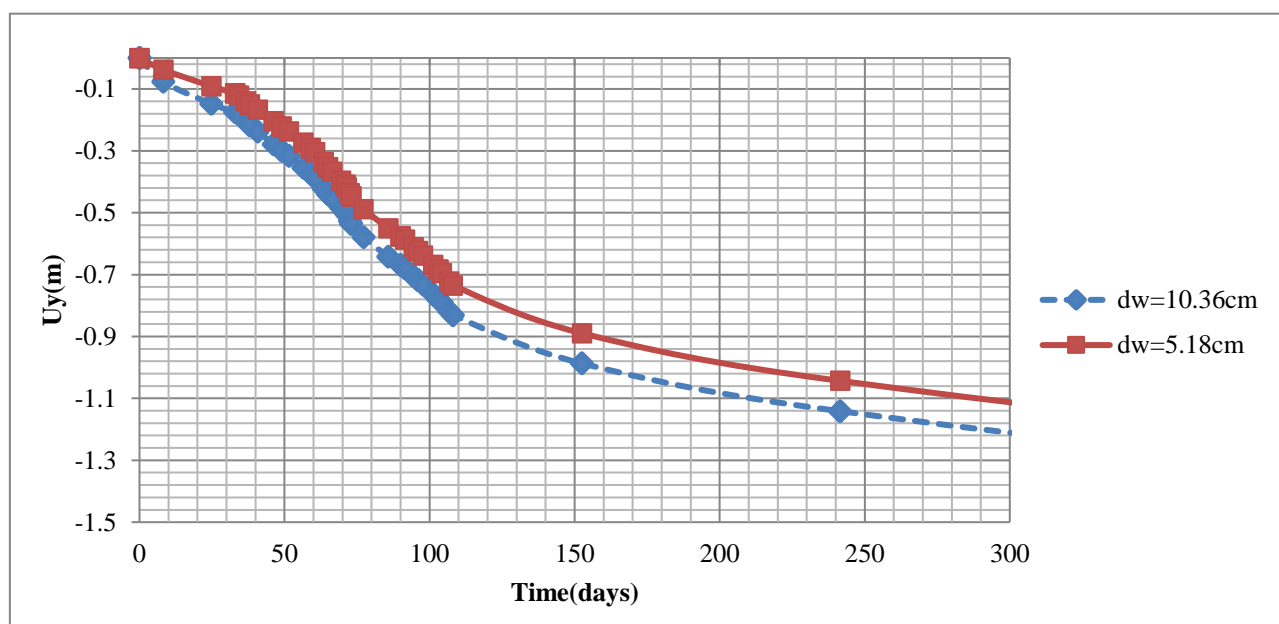


Figure 10. Comparing settlement of the land improved by PVD for two different diameters of 5.18 and 10.36 cm

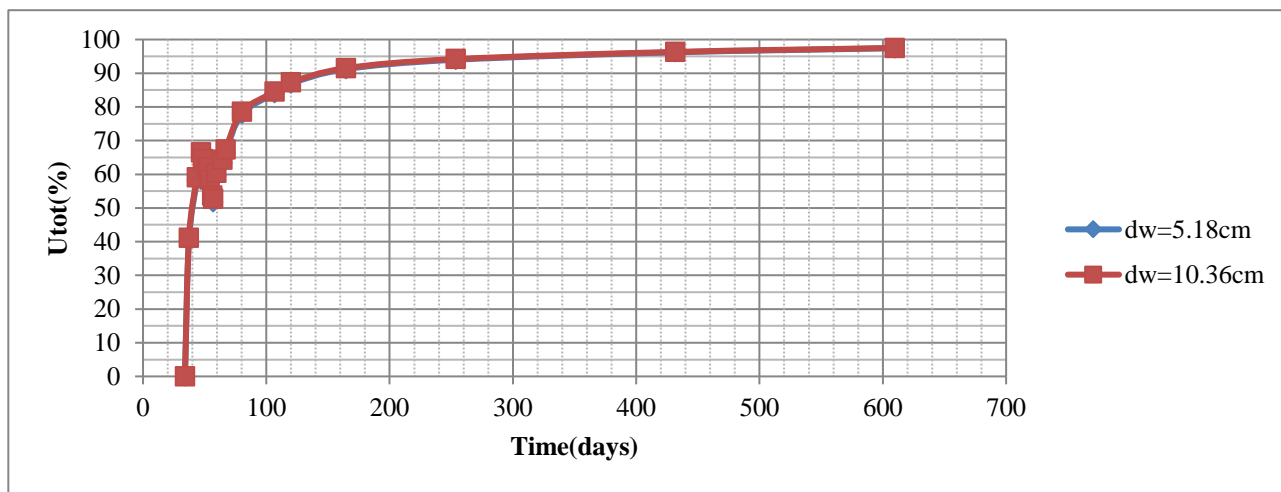


Figure 11. Comparing dissipation of pore water pressure of the land improved by PVD for two different diameters of 5.18 and 10.36cm

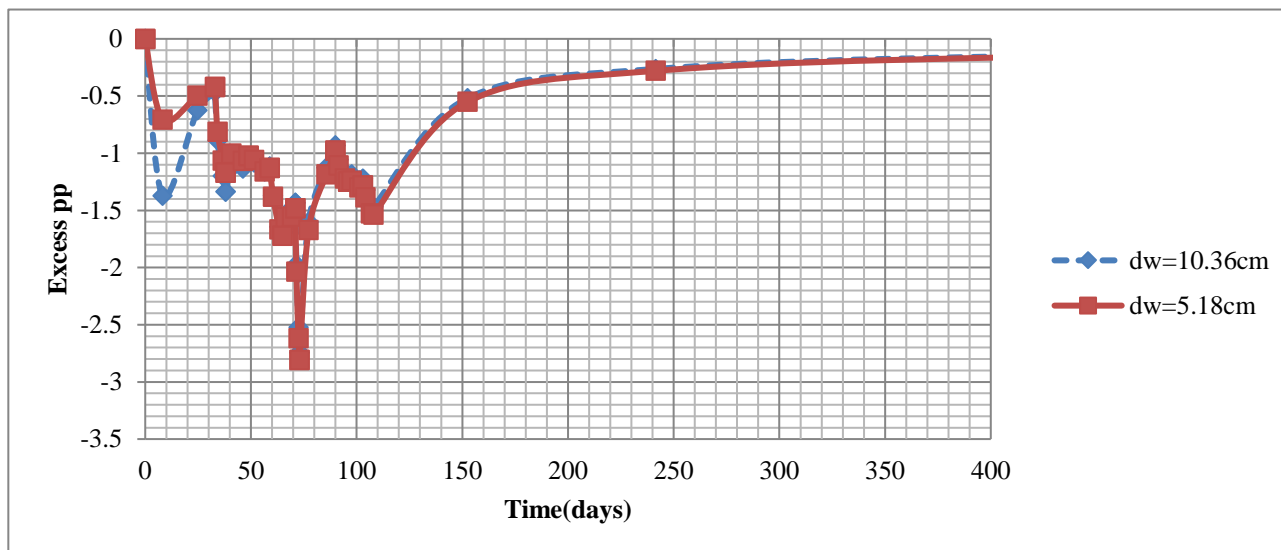


Figure 12. Comparing consolidation rate of the land improved by PVD for two different diameters of 5.18 and 10.36 cm

According to consolidation theory, with increasing PVDs' diameter, their discharge capacity also increases, which increases the consolidation speed. Increasing the rate of consolidation, in turn, allows the layer to settle in a shorter time. For example, after 3 months, PVD diameter increase from 5.18 to 10.36 causes soil settlement's increase from

0.57 to 0.67; and, the excess pore water pressure would be more quickly suppressed. Based on figure 12, despite the change of PVD diameter, curves of average consolidation degree almost match each other; and, this shows a low level of effects of PVD diameter change on expediting procedure of consolidation.

3.3. DISCHARGE CAPACITY VARIATION

According to the table (3) and for fixed installation distance, length, and diameter of PVD, the curve of average consolidation degree, settlement, and dissipation of pore

water pressure in the improved land for various discharge capacities such as 420, 140, and 1260m³/year are shown in Figures 13, 14, and 15.

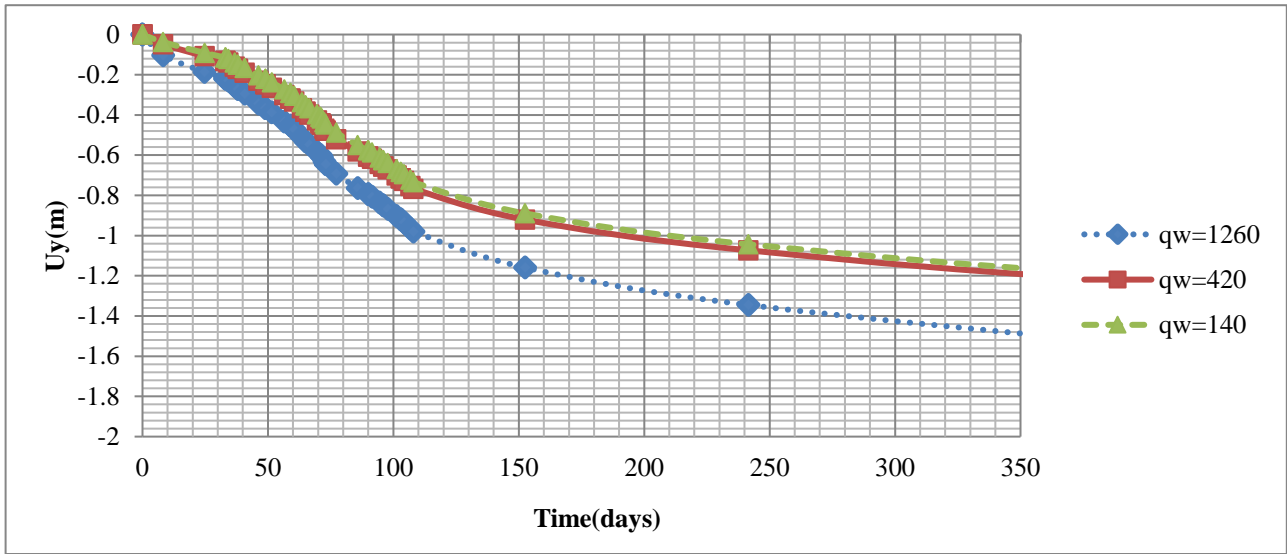


Figure 13. Comparing average consolidation rate for different discharge capacities of 140, 420, and 1260m³/year

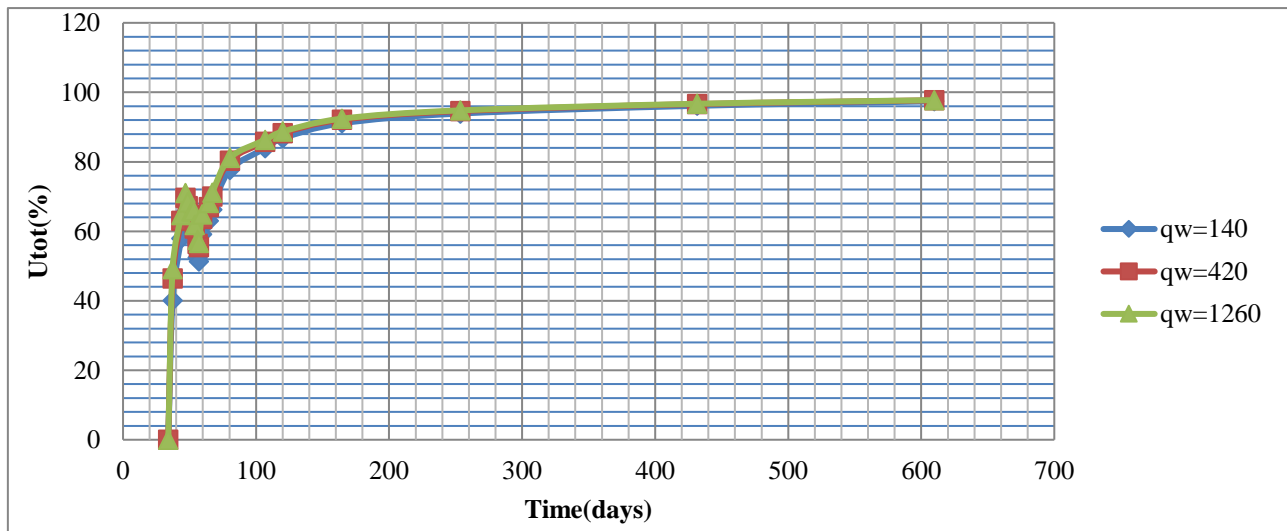


Figure 14. Comparing settlement for different discharge capacities of 140, 420, and 1260m³/year

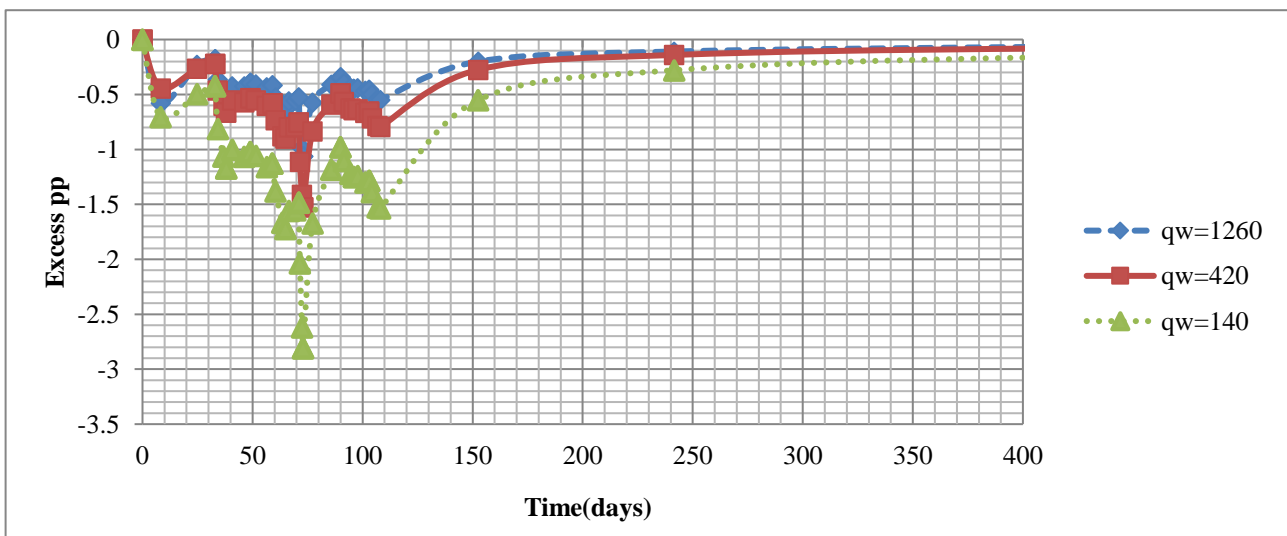


Figure 15. Comparing dissipation of pore water pressure for different discharge capacities of 140, 420, and 1260m³/year

Increasing the discharge capacity alone does not have a significant effect on the rate of layer consolidation. The reason for this is the direct dependence of changes in the rate of consolidation due to changes in the void ratio and the dissipation of the pore water pressure. Therefore, PVD discharge capacity has not much effect on consolidation

rate; an average consolidation degree after 6 months and the increase of discharge capacity from 140 to 1260m³/year would be increased from 83.76% to 84.21%. It has to be noted that, settlement value also has been increased from 0.57 to 0.79 through an increase made in discharge capacity after 3 months.

3.4. LENGTH VARIATION

Considering fixed diameter, installation distance and discharge capacity according to [table 3](#) (curves of average consolidation degree), settlement, and dissipation of pore

water pressure of the land improved by PVD have been compared with each other for three different drainage lengths of 11, 22, and 44m, as shown in [figures 17](#) to [19](#).

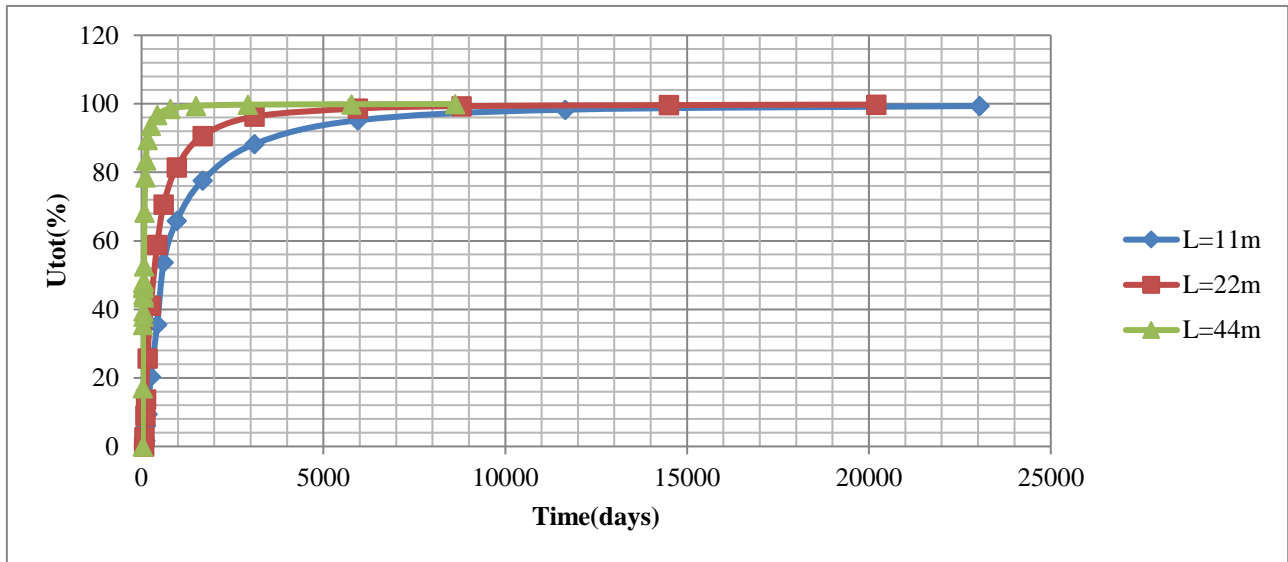


Figure 17. Comparing average consolidation rate for different drainage lengths of 11, 22, and 44m

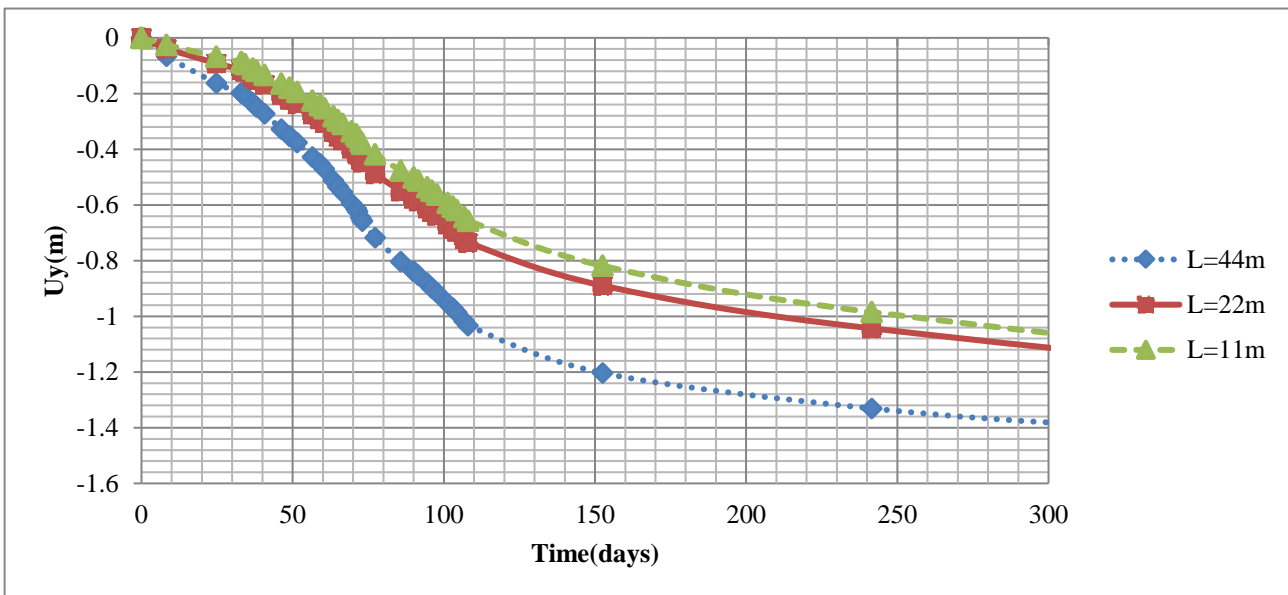


Figure 18. Comparing settlement for different drainage lengths of 11, 22, and 44m

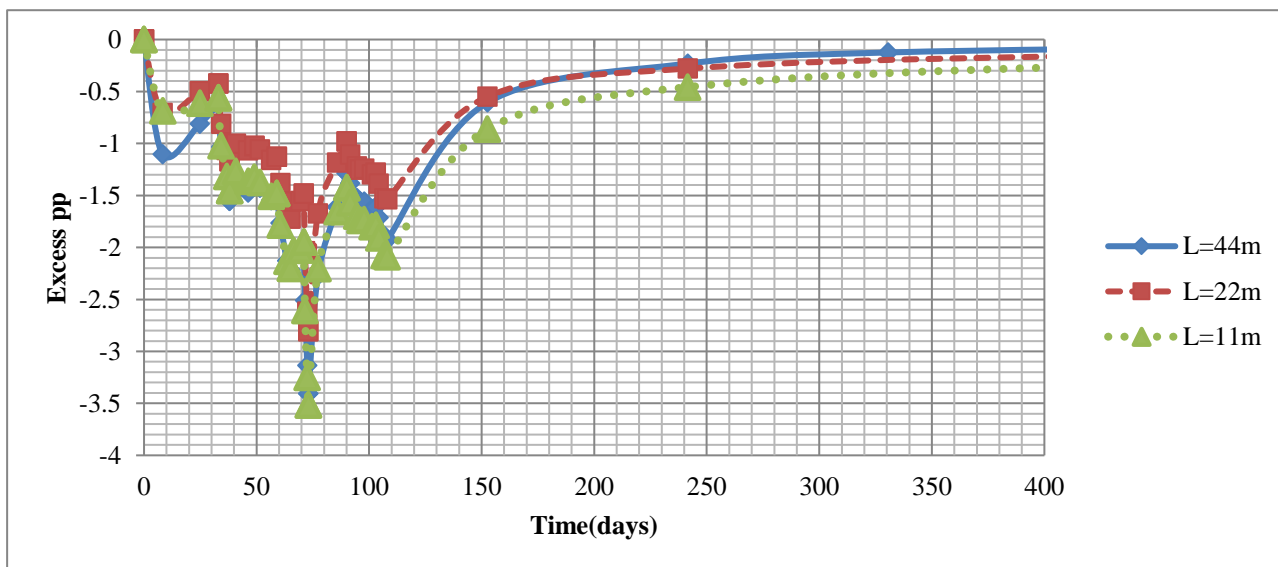


Figure 19. Comparing dissipation of pore water pressure for different drainage lengths of 11, 22, and 44m

As the length of the PVDs increases, the area of the smeared zone increases. As expected, with increasing this parameter, the rate of consolidation speed is strongly affected and must be increased. It should be noted that According to the results of numerical modeling, it becomes clear that PVD length has more effect on expediting the consolidation rate of the improved land, compared to the diameter and discharge

capacity. In [figure 17](#) and with an increase in PVD length from 11 to 44m, the value of consolidation degree U (%) for a period of 6 months increases from 79.21 to 85.77. For more precise study and easier analysis of parameters having an effect on PVD design, computation results are presented briefly in the [table \(4\)](#).

Table 4. Average consolidation degree for various design parameters in the improved condition

$d_w = 5.18\text{cm}$, $q_w = 140\text{ m}^3/\text{year}$, $L_d = 22\text{ m}$				
$S(\text{m})$		1.5	3	4.5
$U(\%)$	3 months	65.12	54.78	46.159
$U(\%)$	6 months	83.76	78.79	74.06
$S = 1.5\text{ m}$, $q_w = 140\text{ m}^3/\text{year}$, $L_d = 22\text{m}$				
$d_w(\text{cm})$		5.18	10.36	
$U(\%)$	3 months	65.12	66.02	-----
$U(\%)$	6 months	83.76	84.21	
$S = 1.5\text{ m}$, $q_w = 140\text{ m}^3/\text{year}$, $L_d = 22\text{ m}$				
$q_w (\text{m}^3/\text{day})$		140	420	1260
$U(\%)$	3 months	67.67	68.04	65.12
$U(\%)$	6 months	84.21	84.68	83.76
$S = 1.5\text{ m}$, $q_w = 140\text{ m}^3/\text{year}$, $d_w = 5.18\text{ cm}$				
$L_d (\text{m})$		11	22	44
$U(\%)$	3 months	59.47	62.25	65.123
$U(\%)$	6 months	79.21	83.76	85.77

4. CONCLUSION

From the literature review that has been studied, the FEM analysis had successfully predicted the behavior of soft clay improved with PVD. It is also confirmed in this study that the consolidation settlement predicted by FEM analysis shows reasonable agreement with the field monitoring

settlement results for PVD treated soft ground [\[28\]](#). It was found that the ultimate settlement predicted from FEM is smaller compared to actual field settlement monitoring results.

In the current paper and to understand consolidation behavior of the land improved by Prefabricated Vertical Drains (PVDs), a parametric study has been performed through usage of PLAXIS 2D finite elements software on clayey soil of Mahshahr Oil Storage and change of PVD design variables such as installation distance, diameter, discharge capacity, and their lengths; and, following results have been obtained:

1- Reducing drain installation distances from 4.5 to 1.5m, and after 6 months from the beginning of preloading, the average consolidation degree has been increased from 74.06 to 83.86%. In other words, the average consolidation degree shows a maximum 10% increase through 67% reduction in distances of drains.

2- Drains' diameters becoming double, average consolidation degree after 6 months has been increased from 83.76 to 84.21; and, it indicates that changes in drain's

diameter have not so much effect on expediting the procedure of soil consolidation.

3- With 9 times increase in discharge capacity of vertical drains i.e. from 140 to 1260m³/year, consolidation degree after 6 months has been increased from 83.76 to 84.21%.

4- Finally, through an increase of drain's length from 11 to 44m, consolidation degree after 6 months has been increased from 79.21 to 85.77%. In other words, with 4 times increase in PVD length, the average consolidation degree has had a maximum change of 5%. In the end, it has to be noted that, from among effective parameters on soil consolidation procedure, the followings have highest to the lowest level of effect on consolidation procedure of soil respectively: change of installation distance; change of length of drain; the increase of diameter; and, increase of PVDs discharge capacity.

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AUTHORS CONTRIBUTION

This work was carried out in collaboration among all authors.

CONFLICT OF INTEREST

The author (s) declared no potential conflicts of interests with respect to the authorship and/or publication of this paper.

5. REFERENCES

[1] Choa V, Bo MW, Chu J. Soil improvement works for Changi East reclamation project. Proceedings of the Institution of Civil Engineers-Ground Improvement. 2001;5(4):141-53. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[2] Long PV, Bergado DT, Nguyen LV, Balasubramaniam AS. Design and performance of soft ground improvement using PVD with and without vacuum consolidation. Geotechnical Engineering Journal of the SEAGS & AGSSEA. 2013 Dec;44(4):36-51. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[3] Paul S, Varghese E, Stephen L. Prefabricated Vertical Drain. International Journal of Engineering Research and Development. 2014;10:50-55. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[4] Basu D, Basu P, Prezzi M. Analytical solutions for consolidation aided by vertical drains. Geomechanics and Geoengineering: An International Journal. 2006 Mar 22;1(1):63-71. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[5] Chen J, Shen SL, Yin ZY, Xu YS, Horpibulsuk S. Evaluation of effective depth of PVD improvement in soft clay deposit: a field case study. Marine Georesources & Geotechnology. 2016 Jul 3;34(5):420-30. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[6] Wang TR, Chen WH. Development in application of prefabricated drains in treatment of soft soils. In General Report. Proceedings of the 3rd Symposium on Weak Ground Improvement using PVDs, Oct 1996 (pp. 13-40). [\[View at Google Scholar\]](#) .

[7] Han J. Principles and practice of ground improvement [Internet]. John Wiley & Sons; 2015 Jun 22. Available from: [d+Improvement.+John+Wiley+and+Sons,+Inc%3B+2015.+&otsfal se. \[\\[View at Google Scholar\\]\]\(#\) ; \[\\[View at Publisher\\]\]\(#\).](http://https://books.google.com/books?hl=en&lr=&id=JTsxBwAAQBAJ&oi=fnd&pg=PA245&dq=Han+J.+Principle+and+Practices+of+Groun</p></div><div data-bbox=)

[8] Hird CC, Pyrah IC, Russell D. Discussion: Finite element analysis of the collapse of reinforced embankments on soft ground. Geotechnique. 1991 Dec;41(4):627-30. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[9] Chen JF, Tolooiyan A, Xue JF, Shi ZM. Performance of a geogrid reinforced soil wall on PVD drained multilayer soft soils. Geotextiles and Geomembranes. 2016 Jun 1;44(3):219-29. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[10] Chu J, Choa V. Quality Control tests of Vertical drains for a land reclamation project. Compression and Consolidation of clayey soils. 1995:43-8. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[11] Rixner JJ, Kraemer SR, Smith AD. Prefabricated vertical drains, vol. I: engineering guidelines. Turner-Fairbank Highway Research Center; 1986 Sep 1. 233. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[12] Asha BS, Mandal JN. Theoretical and numerical modeling of laboratory consolidation of marine clay with natural prefabricated vertical drain. Electronic Journal of Geotechnical Engineering. 2015;20(6):1659-68. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[13] Indraratna B, Bamunawita C, Redana IW, McIntosh G. Modelling of prefabricated vertical drains in soft clay and evaluation of their effectiveness in practice. Proceedings of the Institution of Civil Engineers-Ground Improvement. 2003 Jul;7(3):127-37. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[14] Indraratna B, Aljorany A, Rujikiatkamjorn C. Analytical and numerical modeling of consolidation by vertical drain beneath a

circular embankment. International Journal of Geomechanics. 2008 May;8(3):199-206. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[15] Arulrajah A, Bo M. Finite element modeling of soft soil treated with prefabricated vertical drains. International Journal of Geotechnical Engineering. 2010 Apr 1;4(2):165-79. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[16] Parsa Pajouh A, Fatahi B, Khabbaz H. Numerical Back Analysis of Smear Zone Properties for Vertical Drain Assisted Preloading in Soft Soils. In International Conference on Advances in Geotechnical Engineering 2011. Curtin University. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[17] Voottipruex P, Bergado DT, Lam LG, Hino T. Back-analyses of flow parameters of PVD improved soft Bangkok clay with and without vacuum preloading from settlement data and numerical simulations. Geotextiles and Geomembranes. 2014 Oct 1;42(5):457-67. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[18] Yildiz A, Uysal F. Numerical modelling of Haarajoki test embankment on soft clays with and without PVDs. Geomech Eng. 2015 May 1;8(5):707-26. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[19] Lu M, Sloan SW, Indraratna B, Jing H, Xie K. A new analytical model for consolidation with multiple vertical drains. International Journal for Numerical and Analytical Methods in Geomechanics. 2016 Aug 10;40(11):1623-40. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[20] Bo MW, Arulrajah A, Horpibulsuk S, Chinkulkijniwat A, Leong M. Laboratory measurements of factors affecting discharge capacity of prefabricated vertical drain materials. Soils and Foundations. 2016 Feb 1;56(1):129-37. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[21] Mesri G, Khan AQ. Ground improvement using vacuum loading together with vertical drains. Journal of Geotechnical and Geoenvironmental Engineering. 2012 Jun 1;138(6):680-9. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[22] Chai J, Bergado DT, Shen SL. Modelling prefabricated vertical drain improved ground in plane strain analysis. Proceedings of the Institution of Civil Engineers-Ground Improvement. 2013 May;166(2):65-77. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[23] Chen J, Shen SL, Yin ZY, Xu YS, Horpibulsuk S. Evaluation of effective depth of PVD improvement in soft clay deposit: a field case study. Marine Georesources & Geotechnology. 2016 Jul 3;34(5):420-30. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[24] Venda Oliveira PJ, Cruz RF, Lemos LJ, Almeida e Sousa JN. Numeric modelling of vertical drains: two-and three-dimensional analyses. Proceedings of the Institution of Civil Engineers-Ground Improvement. 2015 May;168(2):144-56. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[25] Indraratna B, Nguyen TT, Carter J, Rujikiatkamjorn C. Influence of biodegradable natural fibre drains on the radial consolidation of soft soil. Computers and Geotechnics. 2016 Sep 1;78:171-80. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[26] Zhou Y, Chai JC. Equivalent 'smear' effect due to non-uniform consolidation surrounding a PVD. Géotechnique. 2017 May;67(5):410-9. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[27] Indraratna B. Recent advances in vertical drains and vacuum preloading for soft ground stabilization, 4th Louis Menard Lecture, Proceedings of 19th International Conference on Soil Mechanics and Geotechnical Engineering, Seoul. 2017; 145–170. [\[View at Google Scholar\]](#) ; [\[View at Publisher\]](#).

[28] Report on Soil Improvement Project for Mahshahr Oil Storages. Sahel Consulting Engineers Institute, 3rd Edition, (In Persian). 2009. [\[View at Google Scholar\]](#) .

[29] A. A. Balkema. Plaxis Manual, Netherland, Version 8.2. 1998. [\[View at Publisher\]](#).